

## **Mathematics (MEI)**

Advanced GCE A2 7895-8

Advanced Subsidiary GCE AS 3895-8

### **Examiners' Reports**

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# 4751 Introduction to Advanced Mathematics

## General Comments

Overall the candidates coped well and it seemed that there were few candidates who clearly were not ready to take the paper. Elegant work was seen from some of the strong candidates. There were some challenging questions to stretch the brightest of candidates, and the grade boundaries reflect this. Although the last part of the last question was omitted by some candidates, examiners felt that this was due to lack of knowledge as to how to proceed, rather than any time problem.

It was evident that surds caused problems to many candidates in questions 11 and 7(i), yet the routine work in question 7(ii) was done well, suggesting that candidates are being well trained to cope with rationalising fractions with surds in the denominator.

The lack of maturity in algebraic manipulation was also evident in many candidates' work, with the last mark in question 5 often lost by 'rooting' individual terms, whilst 13(ii) was often spoilt by 'simplifying'  $x^4 - x^2 - 2$  to  $x^2 - 2$ . The lack of correct use of brackets in their work led to errors by some candidates, for instance in questions 5 and 6, whilst incorrect arithmetic caused problems in question 8, for example, where some candidates could not obtain the correct value for  $y$  when substituting a fractional value for  $x$ .

## Comments on Individual Questions

### Section A

- 1) Finding the equation of the parallel line proved an easy starter as anticipated. Most obtained 3 marks without any difficulty, with very few making arithmetic errors here. Only a small minority used the perpendicular gradient, but even they picked up a mark for substituting (2, 13) correctly into their equation.
- 2) The first part was answered well, although a few answers of  $\frac{1}{4}$  or  $\frac{1}{8}$  or 16 were seen. In the second part, nearly all candidates knew that  $a^0 = 1$ .
- 3) Only about 30% of candidates scored all 3 marks here, with many failing to simplify  $\frac{27}{6}$  in spite of the request. Common errors with the powers were  $(y^4)^3 = y^7$  and  $3^3 = 9$ .
- 4) This was mostly well done. Of those who failed to get both marks, many scored M1 for getting 2.5, but with the wrong inequality. Candidates who began with  $5 < 2x$  were generally more successful than those who started with  $-2x < -5$ .
- 5) There were many poor attempts at changing the subject of this formula, although plenty of good solutions were also seen. Most of the confusion arose from an inability to deal correctly with the square roots at the beginning and at the end. Many made errors in the initial squaring by failing to use brackets, whilst the error in the last step was due to thinking that  $\sqrt{a^2 + b^2} = a + b$ . Follow-through marks for correct work following earlier errors helped candidates to gain partial credit.

- 6) A minority of candidates did this question well; gaining only 2 out of 4 marks was common. A common mistake was not to bracket  $-3x$  and then to square it incorrectly. It was particularly evident in this question that many candidates would benefit from setting out their work more clearly. There were fewer attempts this time to obtain the result by multiplying out instead of by using the binomial theorem, no doubt partly because that would have been so lengthy a process. The special case marks enabled credit to be given to candidates who did not answer the question precisely.
- 7) In part (i) it was evident that only a minority were comfortable with negative and fractional indices and knew that  $81 = 3^4$ , though some worked that out. Many never converted to index form at all. Part (ii) was done well and full marks were common. Candidates knew how to rationalise the denominator and most successfully multiplied out the surds.
- 8) Nearly all candidates knew how to find the intersection of the lines. Over half of the candidates reached  $11x = 7$ , and the majority of these  $x = \frac{7}{11}$ , but a significant proportion then were unable to obtain  $y$  correctly. About a third of the candidates gained all 3 marks.
- 9) Failure to read the question was often seen here as many candidates immediately set about solving the quadratic and ignored what was required in part (i). Those that did attempt to produce an expression for the area were usually successful if they knew the formula for the area of a trapezium and less frequently so if they used a method involving splitting the area or subtraction. One type of mistake was to try to contort an incorrect expression rather than go back to the start. Most did obtain the correct answer for the length of AB as required for part (ii). The equation was usually solved by factorisation, but  $x = 5$  was not difficult to find by inspection. Most realised that lengths had to be positive and so discarded the  $x = -7$  found by factorisation when calculating AB.
- 10) This question was easy to mark and more difficult to do, needing more care and probably time than many gave it. Plenty of good candidates only obtained 1 or 2 marks here, and the full 3 marks was unusual.

## Section B

- 11 (i) Most candidates made a good attempt at this part, coping with the negative signs in the arithmetic. A majority went on to show that the product of their gradients was  $-1$  and therefore concluded that the lines were perpendicular. Some simply quoted the result, e.g. that one gradient was the negative reciprocal of the other or simply that  $m_1 \times m_2 = -1$ , and these were accepted. A small number of candidates calculated their gradients as change in  $x$  / change in  $y$  and thought that they had achieved the correct result by a legitimate method. The alternative method of using Pythagoras was very rarely seen.
- (ii) This part was also attempted well by most candidates, but a large number were unable to cope with the final calculation involving surds, making errors such as  $2\sqrt{10} \times 2\sqrt{10} = 4\sqrt{10}$ , and thus earning just the two method marks. The most common error of method for the area of the triangle was to use the product of the lengths of lines AB and AC; a small number of candidates tried treating the triangle as if it were isosceles, finding a midpoint and attempting to find a height.

- (iii) Not many candidates were able to give clear and concise reasons for the justification for AC being a diameter. However some referred to the perpendicular lines and others based an argument around the fact that triangle ABC was right-angled which enabled them to gain credit. Some other candidates found the distance of point B from the centre of the circle and then compared this with the radius or the diameter of the circle. Attempts at the equation of the circle were generally better and most candidates managed to get some marks here. Finding the centre of the circle was usually done correctly and many candidates demonstrated that they knew what form the equation of the circle should take, either by quoting a general form or by using the coordinates of their centre e.g.  $(x - 6)^2 + (y - 5)^2 = \dots$ . However, determining the radius of the circle and hence completing the equation was done less satisfactorily. Once again it was the arithmetic involving surds which let them down e.g.  $\frac{\sqrt{200}}{2} = \sqrt{100}$ . As a consequence only a few candidates obtained full marks for this part, but marks of 4 or 5 out of 6 were quite common.
- (iv) This part was not done well and the majority of candidates had little idea of a suitable strategy. However, some of the stronger candidates coped well with this part.
- 12 (i) Candidates usually managed to sketch the graph of the cubic in the correct sense, although the quality of the curves drawn was sometimes poor, with 'flicking-out' at the ends. The x-intercepts were commonly well found – although some confused themselves with the non-integer intercept, indicating its position incorrectly on the graph. The x-intercepts were also sometimes given as 1, 4 and 5 as the 5 had not been divided by 2. Many omitted to work out and mark the position of the y – intercept.
- (ii) Again, this was generally well answered. Most candidates chose to multiply out two of the linear factors to obtain a quadratic, usually correctly. Although it was very common for students to omit relevant brackets, they did usually "recover" to obtain a correct, unsimplified form of the answer. Surprisingly many did not collect x terms after multiplying their first pair of brackets, giving 8 terms to be tidied up instead of 6. Candidates who multiplied out all three brackets at once, rather than getting a quadratic factor first, were generally less successful.
- The small number of candidates who opted for the method of showing that 1, 2.5 and 4 were roots of the given cubic were rarely successful. Most managed to show that 1 and 4 were roots, but the arithmetic of showing that 2.5 was a root was beyond these (usually weaker) candidates.
- The (again) small number of candidates who chose to divide the cubic by one of the linear factors were often not convincing enough in factorising the resultant quadratic to score both marks.
- (iii) Part A was generally very well answered by the expected method of substituting 5. A very small number of (usually good) candidates opted for using  $f(5) - 20$ . A very small number of candidates opted to divide at this stage, but usually did not complete the argument that  $x - 5$  being a factor meant that  $x = 5$  was a root.
- Algebraic division or comparison of coefficients method were both popular choices of method offered by candidates in part B. Very often candidates were successful with their efforts, with only a few candidates making occasional sign

or arithmetic errors. It was clear that a small number of candidates did not know where to begin, however, as a few did not attempt this part. Attempts to divide by  $(2x - 5)$  or  $(x - 1)$  were seen occasionally.

It was pleasing in part C to see that most candidates seemed to appreciate that the nature of roots of a quadratic was determined by the discriminant (even though often this was still embedded in the quadratic formula). Most candidates were able to show that the discriminant was negative and give a correct conclusion, but poor arithmetic meant that  $25 - 64$  was often not  $-39$ . Those who did not manage to achieve an answer for part B usually attempted to apply the discriminant to the cubic equation, if they made any attempt here.

- (iv) The word 'translation' was frequently missing from candidates' descriptions of the transformation, with the resultant loss of a mark. '20 down' was common (and accepted), and the majority of those who opted to give a vector were correct – although among the incorrect offerings  $\begin{pmatrix} 0 \\ 20 \end{pmatrix}$  or  $\begin{pmatrix} -20 \\ 0 \end{pmatrix}$  were common.

13 As expected, question 13 as a whole was the question on the paper that candidates found the most challenging, with few gaining marks on part (iii).

- (i) A good number of candidates did score the 3 marks in this part, but many missed the  $y$ -intercept or, more often, the negative  $x$ -intercept.
- (ii) It was very common to see candidates 'simplifying'  $x^4 - x^2$  to  $x^2$ . These candidates scored no further method marks but were able to pick up a maximum of 2 marks if they went on to find the intersections correctly. Those attempting to solve the correct quadratic often gained only part marks since they only took the positive root of 2 or tried to use  $\sqrt{-1}$  for the coordinates.
- (iii) A few of the best candidates got as far as showing that  $k^2 + 8$  was always positive, but in most cases they thought that this was all that was required, and did not go on to show that at least one of the roots for  $x^2$  must be positive in order for there to be a real root for  $x$ . A few candidates tried a graphical approach but their explanations were rarely rigorous enough to gain credit.

## 4752 Concepts for Advanced Mathematics

### General Comments

In general the candidates seemed well prepared for this examination, and the paper was accessible to the overwhelming majority. That said, a surprising number of candidates lost easy marks through a failure to handle routine algebra, and some candidates presented their work so poorly that it was not always possible to tell whether an answer deserved credit or not. For example, was a multiplication sign changed into an addition sign – or vice versa? Centres are reminded of the importance of crossing out work clearly and replacing it clearly. A few candidates made life difficult for examiners by doing a small amount of extra work in the middle of a large supplementary answer book.

Centres are reminded that with a request to “show that”, candidates are expected to work towards the given answer for full credit, rather than *verify* the result (by substitution, for example). A verification approach is unlikely to attract full credit when the demand is “show that”. Many candidates still do not seem to recognise when an answer they have found is clearly not sensible (for example in Q5  $-1 < r < 1$ , Q6 a large positive value for  $d$ , Q12 ( $v$ ) a huge increase in the number of sparrows.)

### Comments on Individual Questions

- 1) The overwhelming majority of candidates scored full marks on this question. A few made a slip with the arithmetic, and lost a mark. The small minority who had extra terms or too few terms did not score. Neither did the small number of candidates who thought it was a geometric progression.
- 2) This question was done very well, with most candidates obtaining full marks. Careless mistakes included the omission of “+ c”,  $2 \div \frac{1}{2} = 1$  and  $3 \div 6 = 2$  (the latter was not penalised.) A few candidates differentiated the second term instead of integrating.
- 3) Most candidates scored full marks. A few slipped up with the arithmetic, and some used the wrong value for  $h$ . Some of the catastrophic errors which resulted in no marks being awarded were  $h = 7.5$ , the omission of the outer brackets and the substitution of  $x$  values instead of  $y$  values.
- 4) (i) Most candidates gained full marks here, but (3, 8), (3, 5/3) and (9, 15) were seen from time to time.
- 4) (ii) Candidates were even more successful with this part. (6, 5) was by far the most common error, although (3/2, 5) was occasionally seen.
- 5) A small number of candidates either could not make a start or thought this was an arithmetic progression. However, most candidates recognised the geometric progression, and many were able to find  $r$  correctly – usually by solving the correct equations simultaneously, but occasionally by trial and error.  $u_n$  was found correctly by most, but those who made sign errors with  $a$  and  $r$  did not earn full credit. Many recognised the appropriate formula for the sum of the first  $n$  terms, but only the best candidates were able to manipulate the brackets correctly and present the right answer.
- 6) Most candidates wrote down two correct equations and solved them correctly to find  $a$  and  $d$ . However, some surprising errors were seen – usually involving division or addition instead of subtraction. A few successfully used a trial and error approach. The last two marks were only obtained by the stronger candidates. The most common errors were to find  $S_{50} - S_{21}$ , or to find  $S_{29}$  (instead of  $S_{30}$ ).
- 7) (i) Most candidates successfully obtained the correct answer. The following errors were commonly seen:  $5 \log x + 12 \log x = 60 \log x$  (or  $17 \log x^2$ ),  $3 \log x^4 = \log x^7$  and  $\log x^5 + 3 \log x^4 = 3 \log x^{5+4}$ .

- 7)(ii)** Most candidates correctly identified at least one of the terms, and most went on to score full marks. Only a few candidates clearly did not understand what was going on, and tried to combine the two terms.
- 8)** A significant minority failed to score any marks at all on this question, either because they did not know how to start, or because their initial step was either  $\cos\theta = 1 - \sin\theta$ , or  $\sin^2\theta = 1 - \cos\theta$ . After a correct initial step, errors in expanding the brackets were often seen, usually resulting in  $\cos^2\theta + \cos\theta = 0$  or  $5\cos^2\theta - \cos\theta = 0$ . At this point many candidates divided by  $\cos\theta$  and missed the roots  $90^\circ$  and  $270^\circ$ . Those who obtained  $101.5^\circ$  often failed to appreciate that there was another root connected with this, or simply added 180 to obtain  $281.5^\circ$ . A surprising number of candidates found  $\cos^{-1}(0.2)$  after earning the first two marks. Only a few then went on to obtain the correct values.
- 9)** Many candidates earned a mark by stating correctly the area of the sector. A few were able to also give the correct formula for the area of the triangle (but a surprisingly large number could not), but only the best were able to deal with  $\sin(\pi/6)$  and equate this with half the area of the sector. There were many fruitless attempts to “fudge” the given answer, often based on using the length  $a$  as the area.
- 10) (i)** Only a few candidates failed to score on this question. Most successfully obtained the equation of the line, and then equated it to the equation of the curve. Marks were then sometimes lost through a failure to solve the resulting quadratic successfully, or for merely stating that at  $x = -\frac{1}{4}$ ,  $y = \frac{1}{4}$ , instead of actually substituting the value in an appropriate formula. Those who adopted a verification approach incurred a small penalty, as detailed in the mark scheme.
- 10) (ii)** This part was done very well indeed. Some candidates lost marks by showing insufficient working in the last part, and a very few thought the gradient of the tangent at A was  $-1/8$ .
- 10) (iii)** Most candidates obtained full marks, but a few equated  $3x + 1$  with  $-2x - \frac{1}{4}$ , and some candidates made mistakes with the algebra and obtained  $y = 7$  (or 1).
- 11) (i)** Nearly all candidates integrated at least two terms correctly to obtain one method mark, and nearly all obtained the third mark for evaluating  $F(3) - F(1)$ . The most frequent errors in the integration were the omission of the denominator of 2 in the third term, or the complete omission of the fourth term. Many made errors with the arithmetic and lost the fourth mark. Many of the responses to the last part were too vague to earn full credit.
- 11) (ii)** There were many very good attempts at this question, although a surprising number of candidates were unable to use the (correct) quadratic formula or complete the square correctly for the third mark. Many candidates lost the accuracy mark through an incorrect simplification of correctly obtained surds. Most candidates were familiar with decreasing functions and were rewarded accordingly.
- 12) (i)** The overwhelming majority of candidates used the given data appropriately and earned the mark for this question.
- 12) (ii)** A disappointingly high number of candidates seemed to be unfamiliar with this standard piece of work, and presented an incorrect equation, or incorrect working to “derive” the correct equation.  $\log P = \log a \times \log 10^{-kt}$  was frequently seen.
- 12) (iii)** This was done very well by nearly all the candidates. Only a few lost the first mark (usually for giving the last value as 4.07 or 4.1, but sometimes all the values were incorrect), and even fewer lost the second mark for correctly plotting their values. A small number of candidates failed to use a ruler and lost the last mark.
- 12)(iv)** Most connected the gradient of the line with  $k$  and many obtained a value within the specified range. Some made a sign error and lost a mark. Nearly all those who attempted the question connected  $\log a$  with the intercept, and obtained a value within the specified range. Only a few candidates interchanged  $k$  and  $a$  and thus failed to score. Even some strong candidates ignored the request for a statement of the equation in the required form and lost an easy mark accordingly.

- 12) (v)** A good number of candidates failed to score. Most candidates who attempted the question realised that a substitution of  $t = 35$  was required, and often went on to score at least one more mark. Some candidates substituted  $t = 2015$  or  $t = 30$ . Most related their answer to 9375 and gave a sensible interpretation, thus earning the third mark even if the second had been lost. A very small number of candidates tried to work directly from their graph, usually because they realised their formula was wrong.

# 4753 Methods for Advanced Mathematics (Written Examination)

## General Comments

Although there was the usual wide range of responses, including many excellent scripts which obtained over 60 marks, this paper proved to be slightly more challenging than its immediate predecessors. In particular, questions 5(ii) and 7(i) were found to be difficult by all but the best candidates, and even question 4, which was a relatively straightforward 3-line proof of a well-known result, caused many candidates to struggle, albeit over only 3 marks. The two section B questions were perhaps seen as more routine and familiar, and consequently, for many candidates, section B outscored section A.

It is worth emphasising to students that they must show valid methods for obtaining correct answers – for example, many candidates gained the correct solution in question 5(ii) but through faulty algebra, and gained few marks. There is also a tendency from some candidates whose algebra is fragile to simplify answers such as in 8(ii) incorrectly. 'Fudging' 'shows' such as in 9(ii) can also lose 'method' marks by introducing inconsistencies into the mathematical argument.

It is noticeable that many candidates' first reaction to a part question is to see it as a new task unrelated to what has gone before, notwithstanding a 'hence' in the question. The point of having longer, linked, section B questions is to encourage strategic thinking – the ability to seek and make connections between different parts. Questions 8(i) and 8(iii), and 9(i) 9(ii) and 9(iv), might serve as useful examples of when to deploy such strategies.

This is the first C3 paper with a printed answer booklet. In general, candidates seem to have had sufficient space to answer the questions in the allotted spaces. If they require more space, it is important that they use additional pages, rather than use space allocated to other questions.

## Comments on Individual Questions

- 1) This question on the chain rule was very well done generally, with most candidates scoring full marks. Occasionally candidates wrote a correct derivative but then made algebraic mistakes in trying to simplify it.
- 2) This modulus question was reasonably well done. Candidates scored 2 out of 4 for getting the correct bounds (1.5 and  $-2.5$ ), and additional 'A' marks for the correct inequalities ( $x \leq -2.5$ ,  $x \geq 1.5$ ). A surprising number of candidates lost a mark for combining the two discrete solution domains in a double inequality, e.g.  $-2.5 \geq x \geq 1.5$ .
- 3) Candidates had mixed success here, although plenty scored full marks. Most recognised the context as an application of the chain rule, and gained a mark for a correct form of this written in appropriate variables. There were some surprisingly incorrect circle formulae offered, and the most common error was to differentiate this as  $dr/dA$  rather than  $dA/dr$ , and substitute  $4\pi$  rather than  $1/4\pi$  into their chain rule.
- 4) There was a disappointing response to this 'direct proof' question. Candidates often seemed to be unclear about how to go about proving a statement, and offered special case triangles such as those with 30, 60 and 90, or 45, 45, and 90 degree angles. Some offered statements such as 'sin = o/h' without defining the angle of the sine or what they meant by 'o' and 'h', which is of course a necessary prerequisite for a formal

proof. Others proved Pythagoras's theorem from the given statement, presenting the argument backwards but without the necessary  $\Leftrightarrow$  signs. Even those who offered a satisfactory proof failed to distinguish between when the statement is true (i.e. for all values) and the range of validity of their proof (i.e. acute angles).

- 5) Part (i) was generally quite well answered. The  $y = e^x - 1$  curve should have shown an asymptote of  $y = -1$ , but this was not required to score the mark. The  $y = 2e^{-x}$  curve required the  $y$ -intercept of  $(0, 2)$  to be shown and  $y = 0$  as an asymptote. Part (ii), on the other hand, was generally poorly done, even by good candidates. The idea of obtaining a quadratic in  $e^x$  proved to be too subtle for most. Many candidates obtained the correct intersection point fortuitously through false arguments, e.g.  $e^{2x} - e^x = 2 \Rightarrow 2x - x = \ln 2$ . The crucial step of recognising that  $e^{2x} = (e^x)^2$  was not sufficiently well known to lead candidates to think of this as a quadratic in  $e^x$ .
- 6) In general, the implicit differentiation was quite well done, either by using a chain rule on the given left hand side, or expanding this first. In the first approach, missing a bracket round the ' $1 + dy/dx$ ' term cost the final 'A' mark, but this method made obtaining the given answer easier. If the second approach was used, candidates were expected to show the correct expansion of  $(x + y)^2$  and obtain the  $2x dy/dx$  and  $2y dy/dx$  terms. The verification of the origin was a stationary point was usually done well.
- 7) Not many candidates scored any marks for part(i) of this question. Marks were given for the upper and lower bounds of the range  $(-\pi + 1, \pi + 1)$ , and a final mark for a correctly defined range (viz  $-\pi + 1 < y < \pi + 1$  or  $-\pi + 1 < f(x) < \pi + 1$ ). Part (ii) was more successful. Finding the inverse function was well done, although some got  $y = \frac{\tan(x-1)}{2}$  rather than  $y = \tan\left(\frac{x-1}{2}\right)$ . The graphical relationship between a function and its inverse are well understood. Candidates needed to show a reasonable reflection in  $y = x$ , and got a mark for showing the  $x$ -intercept  $(1, 0)$ . They lost the mark if the two graphs touched or crossed in the 3<sup>rd</sup> quadrant, or failed to intersect on the  $y=x$  line. Strictly speaking, the inverse graph should not go beyond the vertical in the 1<sup>st</sup> quadrant, but this was condoned.
- 8) Most candidates scored at least 4 out of 7 for part (i), with a reasonable number getting full marks. Integration by substitution was generally well understood, and it was pleasing to see that most candidates included a statement that  $du = dx$  (or equivalent to this). Sometimes the limits changed in the wrong place, but this was condoned. Most expanded  $(u - 1)^3$  from scratch (rather than use the binomial theorem), and there were occasional errors in signs here. The integration was quite well done, the most common mistake being the failure to obtain  $\ln u$  from  $1/u$ .

In part (ii), a small number of candidates unfortunately failed to read the question and differentiated  $x^3/(1 + x)$  instead of  $x^2 \ln(1 + x)$ . The product rule was well understood: errors were in differentiating  $\ln(1 + x)$  as  $1/x$ , and 'simplifying'  $x^2/(1 + x)$  as  $x$ . This unfortunately lost three marks, as the final two depended on their obtaining the correct derivative.

Part (iii) proved to be more challenging. Most candidates obtained the correct 'parts' and put these into the formula correctly. However, many failed to spot the connection between the integral part of this and part (i), and only the best candidates were able to work through the algebra to obtain a correct exact answer. An interesting conceptual error was to put a third of their result of part (i) in square brackets, and thereby adding and subtracting it!

- 9) This question scored higher than question 8, with quite a few accessible marks. Part (i) was well done, with many cases of full marks. However, candidates are prone to muddling negative signs when differentiating  $\sin x$  and  $\cos x$ , and, in trying to 'fudge' the final answer, lost the 'M' mark for the quotient rule, which needed to be consistent with their derivatives.

Part (ii) was a very straightforward 'hence', but not all candidates seem to be used to spotting these sorts of links between parts, and proceeded to integrate  $1/\cos^2 x$  by various false methods.

Part (iii) sometimes suffered from a lack of working. Clearly more was needed than simply stating that 'when  $x = 0$ ,  $f(x) = 1$ '. Verifying  $g(0) = 1$  required candidates to get a correct expression for  $g(x) = 1/2\cos^2(x + \pi/4)$ , which requires a bracket round the  $x + \pi/4$ .

Although most candidates scored over half marks in part (iv), few scored a perfect 8. Candidates were usually well prepared in describing the transformations required. We expect them to use 'one-way stretch' (condoning 'stretch') and 'translation' to describe these: arithmetic operations on the coordinates score no marks. However, translating these into successful graphs proved difficult: they got a mark for the correct asymptotes ( $x = -3\pi/4$  and  $\pi/4$ ), a mark for the correct turning point of  $(-\pi/4, 1/2)$ , a mark for the two curves intersecting on the  $y$ -axis, and a final mark for all this together with a good curve.

The final mark was the preserve of the best candidates. Quite a few tried the integral from scratch, failing to spot the connection between this area and the integral in part (ii).

# 4754 Applications of Advanced Mathematics

## General Comments

This paper proved to be of a similar standard to that set in recent years. The questions were accessible to all and a wide range of marks-from full marks to single figures-was seen. As usual in the January paper most candidates achieved good or high marks. The most disappointing feature was the poor algebra which caused an unnecessary loss of marks for some candidates. The failure to use brackets was part of the problem as could be seen in question 2 and others. The comprehension was understood well and most candidates achieved good scores in that part.

Some candidates did not complete question 8 in Paper A or appeared rushed at this stage. In some cases this was due to using inefficient methods earlier in the paper, but the paper may have been longer than usual.

## Comments on Individual Questions

### Paper A

#### Section A

- 1) Most candidates successfully evaluated the integral using the trapezium rule. Those that failed commonly either failed to evaluate the terms correctly, failed to use the term at  $x=0$ , or misquoted (or misused) the formula. In the second part candidates needed to refer both to the fact that the trapezium rule gave an overestimate in this case and that increasing the number of strips improves accuracy. Often one of these was omitted (or perhaps assumed).
- 2) Almost all candidates found  $t$  in terms of  $x$  correctly and substituted this in the equation for  $y$ . A great deal of poor algebra, both omission of brackets and incorrect signs followed. For example, on the numerator  $1-(1/x-1) = \pm 1/x$  was common. Others failed to clear the subsidiary fractions, either by not trying, inverting fractions term by term or by making multiple errors.
- 3) Most candidates attempted a binomial expansion with power  $-3$  and usually found the correct coefficients. Many could not deal correctly with factorising out the  $1/27$ . Common errors were  $3^3$ ,  $3$  and  $3^{-1}$ . For those who chose the correct factorisation it was disappointing to notice that so many failed to reintroduce it after completing the expansion. In some cases the term in the bracket stayed as  $2x$  even after the factorisation. A more common error, however, was to use  $2x/3$  instead of  $-2x/3$ . There were other algebraic errors after failing to use brackets or signs correctly.  
  
The validity was often correct. Errors included having the signs pointing in the opposite directions, using  $\geq$  instead of  $>$  and expressions such as  $|x| < -3/2$ .
- 4) This question was well answered. Most candidates scored full marks in the first part. The second part was also generally answered well but some candidates tried to use the position vectors of the vertices rather than the direction vectors of the sides. Some failed to see the relevance of part (i) to the area in (ii).

- 5) Almost all candidates correctly stated that  $\sin 2\theta = 2\sin\theta\cos\theta$ . Some used the incorrect double angle formula on the denominator, usually  $\cos 2\theta = 1 - \cos^2\theta$  or  $= 2\sin^2\theta - 1$ . The majority, however, scored all three marks.
- 6) This question was well answered. Most knew the method to find where the line intersected the plane.  
The majority used the correct vectors in the second part although some did not find the acute angle.

**Section B**

- 7) (i) The most common error here was to substitute a value when trying to show that as  $t \rightarrow \infty$ ,  $e^{-2t} \rightarrow 0$ ,  $v \rightarrow 5$ .
- (ii) Only good candidates realised what was required here. Those who realised what was required usually scored well. A few solved the equation rather than verifying as requested. If it was fully correct they could obtain the marks this way, but it was more difficult and time consuming and few were successful. Candidates should be encouraged to verify when requested.
- (iii) The first part was often omitted although there were many candidates who showed correctly that the two differential equations were equivalent. This was attempted both backwards and forwards with success.  
The method of partial fractions was, as usual, well known. A few, however, fiddled their answers to obtain say  $A=1$  and  $B=-1$  in anticipation of the given answer.  
The majority separated their variables correctly. It was disappointing, however, to see some candidates only working with one side and failing to have a  $t$  in their working until reaching the final given answer. There were two common errors here. The first was to integrate  $1/(5-v)$  as  $+\ln(5-v)$  and the second, which was very, very common was to omit the constant of integration. In the latter case candidates are then precluded from obtaining the marks for evaluating  $c$  and establishing the given final result.
- (iv) As in (i), as  $t \rightarrow \infty$  or equivalent was not always seen or used.
- (v) This was almost always correct if the correct values had been obtained in (i) and (iv).
- 8) (i) This was usually answered correctly.
- (ii) Some failed to show that  $CE = BE - BC$ . Most substituted the compound angle formula. The subsequent adding of fractions, cancelling, factorising and the use of  $\sec^2\theta = 1 + \tan^2\theta$  was not often seen. The best candidates found this straightforward but others who attempted this made sign errors and fiddling of results was seen.
- (iii) There was some poor and unclear work in this part. Failing to use  $\sec^2 45 = 2$  and using  $\tan t$  instead of  $t$  or  $\tan\beta$  were common errors. The addition of  $CD$  and  $DE$  to obtain the final given answer was often poor.
- (iv) This question required candidates to 'show' the given result. Stating the result was therefore not enough. Both equations,  $GF = 10\sec\alpha\tan\beta$  and  $GF = \sqrt{2}t$  were given in the question and so the substitution of  $\alpha = 45$  needed to be seen.
- (v) Poor algebra was often seen when showing that  $t^2 = 1 - 1/\sqrt{2}$ . However, the result was usually used correctly to find the value  $\beta$ .

**Paper B**  
**The Comprehension**

- 1)
  - (i) Almost all candidates showed the positions of the 6 guards correctly.
  - (ii) Most candidates realised that the problem arose when  $m$  and  $n$  were both odd. Their justifications, however, were not always correct. Answers such as ' $3 \times 5 / 2 = 7.5$ , there cannot be half a guard' were seen instead of stating that the floor function gave 7 guards instead of the required 8.
- 2)
  - (i)  $2.5 = 2.5$  was often seen.
  - (ii) Often correct, but some incorrectly interpreted the symbol, or left the symbol in the answers.
- 3) This was the least successful question in the comprehension. Those giving a counter-example were the most successful.
- 4) Both parts here were almost always correct.
- 5) This was well answered. Some candidates did not understand what necessary and sufficient meant in these cases. Others did not state which points in the diagram they were using. In part (ii), some said three Cs were necessary or changed the original triangulation but did not refer to it.
- 6) There were many good solutions here with clear constructions and shading but there were also many who did not understand the question.

# 4755 Further Concepts for Advanced Mathematics

## General Comments

Most candidates found the paper accessible and were able to demonstrate knowledge and ability in dealing with mathematical expressions. The overall standard was high, with most responses showing work with good mathematical presentation. Candidates appeared to be scoring more highly in Section A than in Section B on this occasion. It may be that quite a few candidates found themselves working against the clock towards the end of the paper, where there were some instances of no response within parts of both the final questions 8 and 9.

## Comments on Individual Questions

- 1) Mostly well done. It was surprising that many candidates were unable to write down the expansion of  $(x+R)^3$ , but needed to expand and multiply out  $(x+R)^2(x+R)$ . Not all did this successfully. Another common error was to fail to multiply the cubic expansion by  $Q$  completely, usually resulting in the wrong expression for  $S$ , and sometimes for  $P$  as well.
- 2) (i) This was very well done, with most candidates choosing to use the determinant as the appropriate area scale factor. A few tried to transform a particular shape and work out the area of the new shape. This lacked generality and was not carefully explained.  
(ii) Well done by the majority of candidates. There was evidence of confusion in terminology here, where some gave a matrix for the determinant  $\det \mathbf{M}^{-1}$ . Others gave 12 as their answer, not being able to distinguish between  $\det \mathbf{M}$  and  $\det \mathbf{M}^{-1}$ , and another fairly frequent error was to see  $1/13$  or  $13$  for the determinant, also  $1$  and  $0$ .  
(iii) Not many candidates achieved a coherent and concise comment on the value of  $\det \mathbf{M} \times \det \mathbf{M}^{-1}$  which involved the idea of the area scale factor and its role in the transformations. An answer which referred only to matrices was insufficient.
- 3) By far the most popular route chosen through this question was by use of the root relationships  $\sum \alpha$ ,  $\sum \alpha\beta$  and  $\alpha\beta\gamma$ . Some candidates who successfully navigated through the corresponding expressions using the new roots forgot to give an equation at the end, as requested. Others stumbled in the algebra. The substitution method was not quite so popular but usually resulted in a more concise solution but which was still prone to error. The chief mistake was to forget the final term  $+3$  when multiplying to eliminate the fractions. A few candidates used the wrong substitution,  $2w - 1$  and  $w/2 - 1$  were both seen.
- 4) A high proportion of candidates achieved full marks in this question. Very few failed to produce a circle or circles with at least one appropriate radius and with the correct centre. The nature of each boundary was not always clearly defined and in any event it is wise to give a key to define the included and excluded boundaries as there is no universally accepted convention on this.
- 5) This question was also successfully answered by most candidates. Only a few failed to begin with the separation into terms in  $\sum r^2$  and  $\sum r^3$ . These were normally correct. Most candidates then saw the common factors of  $n$  and  $(n + 1)$  and quickly showed the result. Some candidates expanded each term into a polynomial in  $n$  and then had to factorise again. The final step was not always convincing, as the answer was given.

- 6) The answers to this question were variable in quality. Where the candidate had thoroughly absorbed the recommended wording (as has been set out in many previously published mark schemes) there was complete success, but many missed the final two marks through not producing a satisfactory "If...then..." argument as they tried to use their own words. Most candidates stated that  $2^{k+1} + 1 + 2^{k+1}$  was the same as  $2^{k+2} + 1$ , but this was most convincing when the intermediate step  $2 \times 2^{k+1} + 1$  was given as well. A very few candidates thought that they were dealing with the series  $5 + 9 + \dots + u_k$ .
- 7) This was the most confidently answered of the questions in Section B.
- (i) Only a few candidates neglected to give full co-ordinates for the two points.
  - (ii) Only a few candidates neglected to write unambiguously three equations, in full. Nearly all gave the correct horizontal asymptote, but  $y = 1/6$  and  $y = 1/x$  were both seen.
  - (iii) Nearly all candidates gave a clear indication of method, here. Where large numbers were chosen to evaluate the expression in  $x$ , it was more acceptable to see the end of the calculation. An algebraic solution needed careful explanation. Infinity should not be used as a number.
  - (iv) The examiners were looking for a carefully drawn sketch with unambiguous asymptotic approaches and a clear minimum shown in the region  $x < -5$ ,  $y < 0$ .
  - (v) Where the graph was essentially correct, the inequalities followed confidently, and there were not many candidates who wrongly used an inclusive inequality.
- 8) (i) Most candidates found the complex root and many gave good explanations of the reason for another real root, although others were less than coherent. Some of the best answers recalled the nature of the graph of the function.
- (ii) Many earned full marks here. The common errors were to claim the sum of the roots to be  $-1$  or, less frequently, zero.
- (iii) The two popular routes to finding  $a$ ,  $b$  and  $c$  were by means of the root relationships or by multiplying out the factors associated with the four known roots. The former method tested clear thinking owing to the unconventional allocation of  $a$ ,  $b$ , and  $c$  in the original equation, but most candidates negotiated that successfully. There were some mistakes in the expansions in the second method. A few candidates found that one or other of the coefficients was not real, which should have given pause for thought.
- (iv) This was another test of careful thinking. Not many candidates scored both marks in this section, forgetting either that only odd powers of  $z$  would change sign in  $f(-z)$ , or that  $1 \pm j$  would become  $-1 \pm j$ . Numerical answers from (iii) were expected in  $f(-z)$ .
- 9) (i) Maybe because of constraints of time, but many candidates failed sufficiently to justify the terms in the matrix product  $\mathbf{AB}$ , necessary as the answer was given. It was also expected that the factorisation of the resulting diagonal matrix should be explicit, for an easy mark.
- (ii) This was usually well answered but some candidates were evidently uncertain about whether  $(8 + a)$  or  $1/(8 + a)$  should be used with  $\mathbf{B}$ . A few candidates believed that they had already started the next part, and gave a numerical inverse matrix.
- (iii) Mostly well done, but quite a few candidates wasted time by failing to realise that  $a=4$  was needed, until the end of their hard work, and some failed to notice this at all. There were also other numbers used in particular  $a = 0$ .
- (iv) Very few candidates gave an acceptable answer to this. Where it was attempted, many gave "No solutions", or "an infinity of solutions", but not many gave both. "No unique solution" was sufficient for the remaining mark.

## 4756 Further Methods for Advanced Mathematics

### General Comments

After an increase last year, the entry for this paper returned to about the level seen in January 2009. The mean mark was slightly up compared to January 2010, and once again there was a great deal of very good work, with nearly a quarter of candidates scoring 60 marks or more, and only 5% of candidates scoring fewer than 20 marks.

Having said this there were, once again, a number of rather worrying errors, which sometimes appeared even in otherwise good scripts. These included: incorrect assertions and deductions (in Q3, “ $\sqrt{-84}$  is negative”, or “the quadratic does not factorise, hence it has no real roots”; in Q4 “the curve has no turning points, so it is a straight line”); horrendous algebra (in Q1 alone ( $\cos \theta + \sin \theta)^2 = \cos^2 \theta + \sin^2 \theta$ ,  $\sqrt{x^2 + y^2} = x + y$  and  $\frac{1}{2 + \frac{1}{2}x^2} = \frac{1}{2} + 2x^{-2}$  all appeared) and “original” laws

of logarithms (in Q4,  $\ln(4e^t + 3e^{-t}) = \ln 8 \Rightarrow t = \ln 8$ ). In general, candidates were extremely competent when dealing with standard processes, such as finding roots of complex numbers in Q2, finding the characteristic equation of a matrix, finding eigenvectors and using the Cayley-Hamilton Theorem in Q3, and solving the hyperbolic equation in Q4, but the standard of work declined significantly when they were presented with unfamiliar situations, or expected to transfer knowledge from other units. This was particularly evident in Q1(b) which required the binomial theorem from C4, and especially in Q3(iii), where very few candidates were able to exhibit an eigenvector of unit length.

Question 3 was the best done question, with Questions 1 and 4 scoring the lowest. Question 5 (Investigations of Curves) was attempted by only a handful of candidates. There was a little evidence of time trouble, usually affecting candidates who had used very inefficient methods to answer some parts of questions: this was particularly evident in Q1(b), Q2(a)(ii) and (iii) and in some parts of Q4.

Presentation varied from the admirable to the execrable; this time there were a few scripts which were extremely hard to read. Some candidates fitted all their answers into the standard eight-page answer book; others used up to three such books, and there were a few who insisted on presenting graphs and Argand diagrams on separate pieces of graph paper.

### Comments on Individual Questions

- 1) (a) Many candidates converted the curve from polars to Cartesians concisely and elegantly. Others used the given equations to obtain  $x = 2(\cos \theta + \sin \theta)\cos \theta$ ,  $y = 2(\cos \theta + \sin \theta)\sin \theta$  and tried to show that  $x^2 + y^2 = 2x + 2y$  from these, sometimes successfully. A few produced “equations” such as  $r = x^2 + y^2$ ,  $x = \cos \theta$ ,  $y = \sin \theta$ .

The curve was usually recognised as a circle and frequently placed correctly, with all, or most, of the desired information indicated. Only a few candidates rearranged the Cartesian equation to find the centre and radius of the circle, with the majority producing a table of values of  $\theta$  and  $r$ . Some of the sketches were very, very small.

The method for finding the area of a polar curve by integration was well known, with only a very few candidates attempting to integrate  $r$  rather than  $r^2$ . Then, having expanded  $(\cos \theta + \sin \theta)^2$ , most candidates obtained  $\cos^2 \theta + 2 \cos \theta \sin \theta + \sin^2 \theta$ . The identity  $\cos^2 \theta + \sin^2 \theta = 1$  was then commonly ignored, with both

$\cos^2\theta$  and  $\sin^2\theta$  being expressed in terms of  $\cos 2\theta$ . The integration often resulted in a sign error, and sometimes both limits were not used. These faults led to the correct answer being seen relatively rarely.

- (b) The derivative of  $\arctan(\frac{1}{2}x)$  was frequently given as  $\frac{1}{1+\frac{1}{4}x^2}$ .

The intention in the second part here was that candidates should expand  $\frac{2}{4+x^2}$  by the binomial theorem, and integrate the result to obtain a series for  $f(x)$ , considering the arbitrary constant. Only the very best candidates got to the very end, with very few showing that  $c = 0$ . Knowledge of the binomial theorem, and especially how to deal with the 4 in  $(4+x^2)^{-1}$ , was by no means universal. A fairly large number tried to deal with the question by repeated differentiation. If this included valid methods, it was credited, but it rarely did.

- 2) (a) Many candidates spent a great deal of time deriving, rather than writing down, the expressions of  $z^n + z^{-n}$  and  $z^n - z^{-n}$  in simplified trigonometric form, and their answers were not always correct.

Obtaining the given expression for  $\cos^6\theta$  was often done well, but there were many failed attempts. Often the 2s disappeared from, for example,  $z^6 + z^{-6} = 2 \cos 6\theta$ . Candidates who did this often found it convenient to assume that  $2^6 = 32$ . Others ignored the advice in the question and considered  $(\cos \theta + j \sin \theta)^6$ .

Obtaining an expression for  $\cos^6\theta - \sin^6\theta$  caused a great deal of trouble to some candidates, while others used elegant and efficient methods to produce a correct answer in a few lines. The usual approach was to try to obtain an expression as in (ii) for  $\sin^6\theta$ . Candidates started with  $(z - z^{-1})^6$  and expanded it, with varying degrees of success, and often obtained sines of multiple angles rather than cosines when using (i). Others lost a minus sign, or gained a  $j$ , when expanding  $(2j \sin \theta)^6$ . There were many alternative methods, but those who used trigonometric identities alone were rarely successful, and often wasted several pages over the attempt.

- (b) Finding the square and cube roots of  $8e^{j\pi/3}$  was done very efficiently by the majority of candidates, although a little more trouble was experienced in locating the roots in the desired quadrant, and a few weaker candidates failed to divide  $\pi/3$  by 2 or by 3. The Argand diagram was usually adequate, although some forgot to plot  $w$ , and when it did appear, it was sometimes closer to the real than to the imaginary axis.

The method for finding the product  $z_1z_2$  from the exponential forms was well known, and many candidates were able to correctly deduce the quadrant in which the product lay.

- 3) (i) Virtually every candidate knew how to obtain the characteristic equation, although there were a few sign errors. Most expanded by the first row or the first column, and Sarrus' method was also popular. One candidate used elementary row operations to produce a very elegant solution. "Invisible brackets" around the  $1 - \lambda$  were condoned.

- (ii) This was also well done. The quadratic factor was usually obtained correctly, and the absence of real roots deduced, although there were a few careless errors at this stage.
- (iii) The method for producing an eigenvector was well known, although some candidates used  $(\mathbf{M} - \lambda \mathbf{I})\mathbf{v} = \mathbf{v}$  or  $\lambda \mathbf{v}$ . Those who obtained  $3x = 4y$  often went on to give  $x = 3, y = 4$  in the eigenvector. But there were very many correct directions.
- The rest of this part of the question was unpopular with candidates. Only a very few were able to produce a unit vector in the direction of their eigenvector. Most ignored the instruction completely. Slightly more success was obtained with the magnitude of  $\mathbf{M}^n \mathbf{v}$ , although some candidates, seeing  $\mathbf{M}^n$ , were determined to use the result  $\mathbf{M}^n = \mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$ , wasting a lot of time as a result.
- (iv) The Cayley-Hamilton Theorem and its application here were well known, although there were many sign errors, and the identity matrix  $\mathbf{I}$  was sometimes omitted from the answer.
- 4) (i) There were many fully correct solutions to this part. No candidate who used the exponential definitions of  $\sinh$  and  $\cosh$  mixed them up, which was very pleasing. The vast majority of candidates used this method, but four other successful methods were seen, including a very impressive solution using  $r \sinh(x + \alpha)$ , which assisted the candidate in part (ii). Weaker candidates could not obtain, or cope with, a quadratic in  $e^t$ , often inventing spurious laws of logarithms.
- (ii) Most candidates could differentiate  $\cosh 2x + 7 \sinh 2x$ , although a few multiplied by  $\frac{1}{2}$  rather than 2. Then the intention was that, having set this expression equal to 8, candidates would use the answer to part (i). Many did, although they sometimes failed to use the 2 in  $2x$ , but others started again, spending a great deal of time doing so and rarely obtaining the correct answer. The  $y$  co-ordinates were often forgotten. Then candidates were asked to show that there was no point on the curve with gradient 0. Many resorted to exponentials and observed that  $4e^{2x} + 3e^{-2x}$  would never be zero, although often this was just asserted without supporting evidence, and many incorrect statements were seen. Having proved (or not) that there were no turning points, many candidates deduced that the "curve" must be a straight line, and some said as much. However, many did produce the correct shape, although it often passed through the origin rather than  $(0, 1)$ .
- (iii) The majority tried to integrate, often successfully, although a few multiplied by 2 rather than  $\frac{1}{2}$ . Then once again part (i) could be invoked, and often was, although the negative solution was often given as well. Many started again; if errors were made, this often led to nasty quadratics with hideous surds as their roots.
- 5) There were a handful of attempts at this question, of which only one made significant progress past part (i).

## 4758 Differential Equations (Written Examination)

### General Comments

There were many excellent responses to this paper. Candidates seemed familiar with the methods required and were able to employ them appropriately. As is usually the case, most candidates opted to answer questions 1 and 4 together with either question 2 or question 3. The first question was a good starter for almost all candidates and many scored full marks. The other three questions presented some problems, particularly in the later parts. The constant included in the given differential equation in question 3 and the decimals included in the given differential equations in question 4 were the source of some confusion and numerical errors respectively.

### Comments on Individual Questions

#### 1) Second order differential equation

- (a)(i)** This part of the question was almost always answered correctly.
- (a)(ii)** Again, mainly correct answers. Some candidates lost one or two marks through inaccuracy in differentiation or through numerical slips in finding the values of the constant coefficients.
- (b)(i)** This part of the question was, again, almost always correctly answered. A small number of candidates factorised the quadratic equation wrongly.
- (b)(ii)** The key here was to realise that the coefficient of the exponential term corresponding to the given root of the auxiliary equation had to be zero to satisfy the given boundary conditions. Those who failed to spot this were unable to solve to find the other two coefficients.
- (b)(iii)** It was pleasing to see that most candidates were able to solve the exponential equation to obtain  $x = \ln 2$ .

#### 2) First order differential equation

- (a)(i)** The vast majority of candidates who attempted this question gained full marks in this part. Occasionally a constant of integration was omitted.
- (a)(ii)** This part caused a few problems for many candidates. The simplest way to show that  $y$  has a stationary point at  $x = 0$  is to use the given differential equation and substitute the fact that both  $x$  and the first derivative of  $y$  are zero. The result follows immediately. Unfortunately a significant number of candidates differentiated their expression for  $y$  obtained in part (i) and then attempted to solve the resulting equation. This was never a successful approach. The sketch graphs were variable in quality. All that was required was a simple sketch using the information given in this part of the question, namely a curve with a maximum point at  $(0,1)$  and that remained positive for all values of  $x$  whilst tending to zero for large positive and negative values of  $x$ .
- (b)(i)** This straightforward example of the use of Euler's method posed few problems for candidates. Occasionally, there was some confusion with the estimation of the value of  $y$  for  $x = 0.3$  being offered as the estimate for  $x = 0.2$ .

**(b)(ii)** Candidates invariably integrated correctly using an integrating factor, but then had difficulty in using the limits of 0 and 0.2 appropriately. Often on the left hand side of their integrated expression,  $y$  was taken out as a common factor before the limits were applied.

**3)** First order differential equation

**(i)** Candidates clearly knew how to attempt this question and almost all found the correct complementary function. The search for the particular integral, however, was not often successful. Candidates became confused between the constant coefficients in their trial particular integral and the constant  $k$  in the given differential equation.

**(ii)** The method here was applied correctly to what was usually an incorrect general solution from part (i)

**(iii)** Many candidates continued with the method they had used in part (i) of this question. Others used the integrating factor method and this latter gave a neat solution.

**(iv)** Very few candidates were able to use the approach suggested in the question. They spotted the similarity between the two differential equations, with  $k = -2$ , but did not know what to do with this information. Those who opted for the "otherwise" approach of solving the new differential equation from scratch were usually more successful.

**4)** Simultaneous differential equations

**(i)** Solutions were almost always fully correct.

**(ii)** Again, the vast majority of candidates obtained full marks for a correct general solution for  $x$ .

**(iii)** It was pleasing to see that almost all candidates employed the correct approach here, differentiating their expression for  $x$  and using it in the first of the given differential equations to find  $y$ . Unfortunately the decimal quantities involved seemed to lead to more numerical errors than might have been expected. The numbers 0.1, 0.2 and 10 together with some dubious use of brackets led to many incorrect expressions for  $y$ .

**(iv)** By this stage, the number of correct solutions was decreasing fairly rapidly with some extra confusion coming from the manipulation of four constants, and an uncertainty as to which were to appear in the final expressions.

**(v)** There were some excellent solutions to this part of the question, from candidates who had negotiated carefully the earlier parts of the question and now knew how to solve the equations involving a sine term and a cosine term. It was unfortunate that a few who had done well thus far used degrees rather than radians. Candidates who had made errors earlier in the question could potentially have gained credit, but the majority failed to recognise a method for extracting  $t$  from their equations. The easiest approach was to write  $\sin/\cos$  as  $\tan$ . An alternative was to rewrite their expressions in the form  $R\sin(x + \alpha)$ .

# 4761 Mechanics 1

## General Comments

Most of the candidates were able to make some progress with every question and many made excellent attempts at all of them. Even the strongest candidates were challenged by answers that required an account of the reasoning used; for instance, many candidates failed to convince when attempting to justify their claim that the force in the rod in Q5 was a thrust. It was pleasing to see many candidates able to argue convincingly in Q4(ii) that the velocity was never zero.

The great majority of the candidates showed that they had covered the syllabus thoroughly and many demonstrated the ability to apply their knowledge and skills effectively.

Candidates should either write their answers in the space provided in the Printed Answer Book or write them clearly labelled on supplementary sheets.

## Comments on Individual Questions

- 1 (i) Almost all of the candidates completed the graph correctly.
- (ii) Many of the candidates had a clear idea of what to do to find the acceleration. A higher proportion of those who tried to find the gradient of the line were successful than those who tried to use the suvat results, but a common mistake was to give 2 instead of  $-2$  as the answer. A surprisingly large number of candidates took the initial speed to be  $15 \text{ m s}^{-1}$ , despite the value being given in the text as well as on the graph.
- (iii) Quite a few candidates got this wrong but most knew what they were doing. Those who tried to find the signed area under the graph were more likely to get the right answer than those who used the suvat results. Some candidates clearly did not know what 'the area under the graph' means. Common errors were: taking the lowest velocity to be  $-5$  instead of  $-6 \text{ m s}^{-1}$ ; not counting the areas below the time axis as negative and so finding the distance travelled instead of the displacement; finding the displacement from  $t = 0$  to  $t = 10$  correctly using suvat results and then subtracting the areas 'under' the graph from  $t = 7$  to  $t = 20$ .
- 2 (i) Most candidates knew what to do but quite a few got into a muddle with the signs of the components of the force of magnitude  $8 \text{ N}$ . A few candidates failed to give their answers correct to 3 significant figures.
- (ii) Many perfect answers, the most common error being to find the angle with the horizontal instead of the upward vertical.
- 3 (i) A few candidates could not do this; quite a few gave expressions that were not distances or displacements.
- (ii) It seems that most candidates had no problem with visualising the situation and formulating the required (and given) quadratic equation. Some argued using distance and others using displacement. However, quite a few candidates got themselves into a muddle and, having found that the two cars had travelled distances  $8T \text{ m}$  and  $2T^2 \text{ m}$  towards each other from positions  $90 \text{ m}$  apart when they collided after  $T \text{ s}$ , could not write down  $8T + 2T^2 = 90$  or an equivalent expression; some gave up after several tries and others made no attempt. I

can only think that this was due to their not being used to formulating equations from given information. Most candidates correctly solved the quadratic but a few lost the final mark as they gave both the positive and the negative roots as answers.

4 In this question as well as Q2, a few candidates seemed unaware of the meaning of a vector expression and the comments below refer only to the candidates who did. Some candidates showed awareness in only one of these questions.

- (i) Almost all candidates found the position vector of P and its bearing from O.
- (ii) Only a very few candidates failed to use calculus in some way and most obtained the correct vector. Many candidates correctly stated that the vector could not be zero as its  $i$  component was always 8 and so it must always have a speed of at least  $8 \text{ m s}^{-1}$ . A few candidates only claimed that the vector was not zero when  $t = 0$ , which is not a complete argument.
- (iii) Most candidates knew they should differentiate their velocity vector and did this accurately. Almost all of these candidates then found the required value of  $t$ . Some candidates who did not write down the acceleration in vector form (which was acceptable) then made no reference at all to the  $i$  component (which was not).

5 (i) Almost all of the candidates knew how to find the deceleration correctly. Most did this directly and accurately using  $v^2 = u^2 + 2as$  but some went via finding the time and falsely used the zero acceleration result  $s = ut$ .

More candidates tried to find  $F$ , used Newton's second law on the complete system with mass 18 kg than considered separate equations of motion. In both methods candidates made sign errors. Many using the complete system method used  $F$  instead of  $2F$  but on the whole those using this method made fewer mistakes than those producing two equations.

- (ii) There were some excellent answers to this part but not many candidates produced a correct solution. The most common problem was with sign errors. The arguments used to establish whether the rod was in tension or compression were often falsely based on wrong general principles (for example, the boxes are decelerating so the rod is in compression) or incomplete arguments.

6 There were many excellent solutions to all or almost all parts of this question. Unusually, some good answers came from candidates who had not done particularly well overall. It seemed that there were fewer instances than in recent series of incorrect resolution, confusion of mass and weight, and misunderstanding of the term normal reaction. Apart from the small number of candidates who took Newton's second law to be  $F = mga$  or  $F - mg = ma$ , wrong answers mostly came from sign errors made either in the formulation of the equations or during manipulation or from omitted forces.

- (i) Most candidates knew what to do and obtained the correct answer.
- (ii) Again, most candidates knew they should resolve horizontally and did so accurately.

- (iii) The few candidates who resolved other than parallel to the plane usually omitted the component of the normal reaction.
- (iv)(A) A lot of candidates failed to produce a correct diagram. The most common errors were: omitting the normal reaction; omitting an arrow or label; introducing another friction force down the plane; putting in a force and its components (without indicating in some way that the components were not additional forces).
- (iv)(B) Most candidates started this part from first principles but a pleasing number saw the short cut and wrote down  $\cos\theta =$  their answer to (iii).
- (iv)(C) I thought that, compared with recent examination series, fewer candidates did not use a proper definition of 'normal reaction'; however, this remained the most common error, especially using the false definition of normal reaction as being the component of the weight perpendicular to the plane.

7 A fairly common mistake was to take the origin to be at ground instead of platform level; this mistake made it harder to establish some of the results and impossible to find the given cartesian trajectory equation in part (v).

- (i) Most candidates correctly wrote down the components of displacement at time  $t$ .
- (ii) Many of the candidates saw what to do. Many who had the origin on the ground in (i)(B) used correct their equations in this part. Candidates who argued that  $y = 0$  when  $t = T$  tended to do better than those who argued that  $T$  is twice the time it takes for the ball to reach its highest point. Quite a few candidates fudged the result by using  $v = u + at$  with  $a$  stated to be 4.9 instead of  $t = \frac{1}{2} T$ . Quite a few candidates did not use the horizontal displacement to obtain their second equation but instead simply gave a different form of the equation obtained by considering the vertical displacement (this was often different from the first equation because of a derivation or manipulation error).
- (iii) Many candidates attempted to do the right thing, many producing the correct horizontal displacement equation in this part when they had no equation or the wrong equation in part (ii). A common error was for candidates to fail to show that their answers agreed with the correct answer to 3 significant figures. No candidate who tried to show that the use of  $u = 12.0$  gave consistent results in the two equations could establish the accuracy of the result. There were many good answers to this part.
- (iv) Many candidates made mistakes in this part which required, perhaps, the clearest thinking on the paper. A common mistake was to equate  $y$  to 0 or even  $+ 2$  instead of  $- 2$ . Most candidates who got started on this part realized that they should use the quadratic formula and did so accurately. There were many clear, concise and correct solutions.
- (v) Most candidates who attempted this part knew how to establish the cartesian equation of the trajectory and did so well. The last request defeated many candidates, who didn't realize that they just need to use the result  $y = 0$  when  $x = 10$  (or  $y = - 2$  when  $x$  takes the value  $10 +$  their answer to part (iv)). There were many nice, concise answers.

## 4762 Mechanics 2

### General Comments

The standard of the responses to this paper was generally very good and often excellent. Most candidates were able to make a reasonable attempt at most parts of the paper. The standard of presentation varied, as usual. There were many neat, well-ordered answers but there were also a significant minority in which it was difficult to track the candidate's train of thought. The latter almost invariably led to inaccurate work and a loss of marks. It was pleasing to note, however, that most candidates seemed to have a good grasp of the mechanical principles involved, even if they were not able to carry their solutions through with accuracy.

### Comments on Individual Questions

- 1) Many candidates scored high marks on this question and almost all candidates showed a pleasing understanding of the principles involved and an ability to apply them appropriately.
  - (i) This posed few problems for the majority of candidates. Some candidates, however, having found the value of the maximum frictional force, failed to go on to attempt to show that the block *A* remains at rest.
  - (ii) Almost all candidates showed a good understanding of the principle of conservation of momentum and of Newton's experimental law and many scored full marks. Candidates must realise that sufficient working must be shown when obtaining an answer given in the question. In this case, some evidence was expected of how the answer of 8.4 was obtained from the simultaneous linear equations. A few candidates found the velocity of *B* by assuming the given value for the velocity of *A*. Such attempts gained little credit.
    - (iii) Almost all candidates were able to find the magnitude of the impulse, but a significant number failed to specify its direction.
    - (iv) There are three common methods of approach to this part of the question and candidates showed no particular preference for one rather than the others. In each method, however, a significant number of candidates omitted one of the two forces acting on *A* parallel to the slope.
- 2) This was the least well-answered question on the paper. Candidates were usually aware of the principles of mechanics that were involved, but were often not able to apply them appropriately to the situation described in the question.
  - (a) Many candidates were able to apply the principle of conservation of linear momentum appropriately here, but a significant number seemed to ignore the first sentence in the question when applying conservation of energy. It was common to see candidates using a combined mass for *B* and *C* with a single combined velocity. This resulted in few marks being awarded. Of those who were successful in calculating the speeds of *B* and *C*, many lost a mark by failing to indicate the directions.
  - (b)(i) There were many pleasing answers, showing a sound understanding of the use of the work-energy equation. Any loss of marks was usually due to sign errors.

- (b)(ii)** This part of the question was poorly done by many candidates. The solutions offered showed much confusion in understanding. The formulae for work done as " $Pt$ " or as " $Fd$ " and for power as " $Fv$ " often appeared to be used randomly in incoherent solutions. Method marks were awarded wherever possible. A major stumbling block was to assume that the vertical height of 20m gained by the car was in fact the distance travelled by the car along the slope.
- 3)** Many excellent answers to this question were seen.
- (i)** This was almost always correctly answered.
- (ii)** Again, usually correct. The only source of error was to use a circular argument that assumed the results in order to prove them. Such errors gained no marks.
- (iii)** Again, usually correct. Any loss of a mark was through a failure to label the internal forces on the rods, needed to indicate their equality in magnitude for each rod.
- (iv)** There were many excellent solutions to this part of the question. Candidates who adopted a methodical and clearly structured approach often gained full marks. The most common errors were sign errors in resolving the forces at a pin-joint. A minority of candidates seemed to have no idea what was required. This is surprising because this was a fairly standard question on this topic.
- (v)** The simplest approach here, followed by most candidates who attempted this part of the question, was to resolve in two directions at  $C$  and obtain the same results as previously. Those who attempted to explain in words, along the lines that changes at  $D$  did not affect what happened at  $C$ , rarely presented a complete argument.
- 4)** The standard of the solutions to most parts of this question was good; the request to find the centre of mass of the cross-section of the prism was particularly well-answered. Success in the last two parts of the question depended on the candidate's ability to choose the sensible point about which to take moments.
- (i)** This part of the question was almost always correctly answered.
- (ii)** About half of the candidates obtained full marks here. Others understood the essence of the method, but lost marks either through sign errors when taking moments or through a lack of evidence in reaching the given result.
- (iii)** The mechanics involved here was less of a problem to the candidates than a clear understanding of how to work with inequalities.
- (iv)** The majority of candidates realised that the prism would tip about the point  $O$  and consequently took moments about  $O$ . Errors crept in with finding the distances involved.
- (v)** Candidates who realised that the prism would tip about the point  $D$  and so took moments about  $D$  usually gained full marks. Many candidates, however, attempted to take moments about  $O$  and omitted the reaction at  $D$  between the prism and the horizontal plane. Such attempts rarely gained any marks.

## 4763 Mechanics 3

### General Comments

Most candidates were able to demonstrate their competence in many of the topics being examined. The work on this paper was generally of a high standard, with half the candidates scoring marks in the 60s and 70s (out of 72).

### Comments on Individual Questions

- 1) (i) The dimensions of force and density were almost always given correctly, but a large number of candidates gave the dimensions of angular speed as  $\text{LT}^{-1}$  instead of  $\text{T}^{-1}$ .
- (ii) Almost all candidates established the dimensions of breaking stress correctly. Just a few wrote  $\frac{\text{MLT}^{-2}}{\text{L}^{-2}}$  instead of  $\frac{\text{MLT}^{-2}}{\text{L}^2}$ .
- (iii) About half the candidates were unable to convert the units successfully. By far the most common error was to multiply by the reciprocal of the correct conversion factor.
- (iv) The method for finding the indices in the formula was very well understood, and most candidates completed the work accurately.
- (v) Most candidates knew how to find the breaking stress of aluminium, and many obtained the correct answer.
- (vi) This was also answered well. Some candidates did not realise that the value of the dimensionless constant  $k$  would be the same in the new system of units, usually assuming that  $k=1$ .
- 2) (a)(i) Most candidates gave equations for vertical equilibrium and horizontal acceleration. However, about a third of the candidates failed to obtain the correct answers; the normal reaction was often omitted completely, and very often put in horizontally or at right angles to the rope.
- (a)(ii) This situation (with no contact force) seemed to be much more familiar to the candidates, and most answered this part correctly.
- (b)(i) Almost all candidates applied conservation of energy to obtain the given result convincingly. Just a few wrote down  $v^2 = u^2 + 2as$  without any explanation, and this did not earn any credit.
- (b)(ii) The radial equation of motion was very often given correctly, although sign errors were fairly frequent. It was then necessary to substitute for  $\cos\theta$  from the result in part (i), but a surprisingly large proportion of candidates failed to do this.

- (b)(iii)** Almost all candidates knew that P leaves the surface when  $R=0$ , and the value of  $u$  can then be found immediately from the expression in part (ii). Many candidates who had not completed part (ii) were nevertheless able to answer this part independently and correctly.
- 3) (i)** Almost all candidates found the tension correctly, although a few confused stiffness with modulus of elasticity.
- (ii)** Most candidates also obtained the natural length of the bottom rope correctly.
- (iii)** Most candidates realised that they should apply Newton's second law to set up the equation of motion. However, the expressions for the tensions were very often incorrect and there were many sign errors. A significant proportion of candidates omitted this part altogether.
- (iv)** The maximum acceleration was usually found correctly; some candidates appeared to be finding the maximum speed instead.
- (v)** This was also answered well. Most candidates used  $v^2 = \omega^2(A^2 - x^2)$ , and a fairly common error was to take  $x=0.6$  or  $x=3.8$  instead of  $x=1.6$ . Some preferred to use displacement-time and velocity-time equations.
- (vi)** Most candidates found the position of the block after 5 seconds (although some worked in degrees instead of radians). However, only a minority could work out the distance travelled to get there.
- 4) (a)** Almost all candidates knew the method for finding the centre of mass of a solid of revolution. There were some careless slips when evaluating the definite integrals, and some candidates appeared to be working with  $y^2 = x^2 + k^2$  or  $y^2 = x - k$  or  $y^2 = (x^2 - k^2)^2$  instead of  $y^2 = x^2 - k^2$ .
- (b)(i)** The centre of mass of the lamina was found correctly by most candidates; the only common error being omission of a factor  $\frac{1}{2}$  from the y-coordinate.
- (b)(ii)** The method was well understood, and the angle was very often found correctly.

## Chief Examiner's Introduction to Statistics Reports

Two general matters, to which some attention is also drawn in the individual subject reports, are worthy of mention in a general introduction, as they apply to all the Statistics modules.

First, advice was circulated several months ago concerning the issue of numerical accuracy of final answers, in particular to the practice of some candidates of gross over-specification in this regard. As an example, this would refer to the quotation of the value of a test statistic as, say, 2.18735693762 merely because this is the number that happened to appear on the candidate's calculator. This shows a complete lack of understanding of statistical practice and, indeed, of basic concepts of numeracy. In the current round of examinations, accuracy marks (but not method marks) were normally withheld in such cases. The earlier advice had explicitly stated that this would occur, and it will continue in future rounds. This is of course different from the desirable practice of retaining sufficient accuracy in intermediate calculations to avoid problems resulting from premature rounding.

Secondly, there are many references in the individual subject reports to the importance of securely stating hypotheses when conducting statistical tests. In future rounds of examinations, candidates will be expected to state their null and alternative hypotheses even if this is not explicitly asked for in the question. In many cases, this can sensibly and compactly be done in the usual notation of the subject, for example " $H_0: \mu = 25$ ;  $H_1: \mu > 25$ ", but it would be expected that any parameters appearing in those statements are themselves briefly but adequately defined verbally. In the example, this might be achieved by adding "where  $\mu$  is the population mean". There is no objection to hypotheses being stated verbally (for example "the null hypothesis is that the population mean is 25 (cm) and the alternative hypothesis is that it is greater"), but candidates must be careful to be precise in their wording (notably, explicit use of the word "population" will often be necessary for full marks to be awarded).

# 4766 Statistics 1

## General Comments

The level of difficulty of the paper appeared to be appropriate for the candidates. There were fewer cases than in previous sessions where candidates scored most of their marks on one or two topics only and all the questions seemed accessible to any reasonably prepared candidate. The more able candidates scored well throughout the paper with the exception of question 6(iii) (the answer  $0.93^3$  was common), and 7(v) (the new variance often being given as  $1.15 \times 33.25^2$ ). The less successful candidates often gained the majority of their marks from question 4 and the more straightforward parts of the probability tasks in questions 5, 6 and 8. No candidates appeared to have problems in completing the paper in the time available.

Most candidates supported their numerical answers with appropriate explanations and working. However there were a small number who left out the essential steps in a solution. For example there were a few who, in the hypothesis test, reached a correct probability of 0.0867 or 0.9133 and then failed to compare explicitly with 0.05 or 0.95, thus losing three marks. Some candidates did not state any probabilities to justify their critical region. In the past this omission has been treated generously, but in future candidates who fail to do so can expect to receive very few if any marks. A similar loss of three marks was incurred by those who just wrote an answer of 0.8 without any working in question 6(iii). Arithmetic accuracy was generally good although there is still evidence of candidates not being proficient or sensible in their use of calculators. In particular many candidates did not check their answers to question 7(ii) and 7(iii) with their calculator. The use of point probabilities still occurs in the hypothesis test, not only by the weakest candidates, although rather less frequently than in previous series. There was also an apparent lack of understanding of independent events, with only the more able candidates being able to give a convincing argument as to why the two events were dependent in question 2.

The over-specification of answers was prevalent in question 7. Inevitably the average and more able candidates were penalized more heavily by the loss of an accuracy mark as often the weaker candidates had already made some error in their working. It did appear that few candidates were aware of the necessity to consider how many figures should be specified. Often the answer to an estimated mean was given to 5 or more significant figures. On this occasion only, a maximum of 2 marks were deducted in a question for over-specification. However, in future series, over-specification will be penalised every time it occurs. (Please see the 'Note on accuracy in Statistics modules' contained in the Chief Examiners' report for June 2010).

Presentation was generally good. Fortunately only a small minority of candidates attempted parts of questions in answer sections intended for a different question/part! On a practical note most students are more aware than previously on how to use the answer booklet effectively i.e. using heavier pens, starting at the top of the box, clearly identifying final answers etc. Few candidates needed to use an additional answer book although for those that did, the use of additional sheets of graph paper caused problems with seeing the graphs. Candidates should draw their graphs boldly so that they can be viewed on screen clearly.

## Comments on Individual Questions

- 1) (i) Most candidates achieved full marks with the most common mistake being to leave the answers as 96 and 102.
- 1) (ii) Almost all candidates could identify a positive skew correctly.
- 2) (i) Most candidates answered correctly. Occasionally a wrong answer of  $3/16$  was seen.

- 2) (ii)** In contrast this part was only correctly completed by about one third of the candidature. Although many could quote a correct test for independence eg  $P(A \cap B) = P(A) \times P(B)$  a substantial minority attempted vague arguments about being "connected". For those using probabilities, calculating  $P(\text{even} \cap <10)$  from the table seemed difficult, and many, having focussed on the "even" rows, calculated  $P(<10|\text{even})$  instead. Those who attempted some form of conditional probability, e.g. comparing  $P(<10|\text{even})$  with  $P(<10)$ , usually failed to produce a correct argument, even if they managed to state the two probabilities which they were comparing. Rarely did they find both of them correctly. Some good solutions lost a mark due to the lack of a conclusion.
- 3) (i)** Almost all candidates answered correctly but a very small number used permutations rather than combinations.
- 3) (ii)** The majority of candidates answered correctly but a number added rather than multiplying or evaluated  $\binom{23}{6}$ .
- 3) (iii)** Relatively few correct answers were seen in this part. A product of fractions leading to 0.017 was quite often seen as was a final answer of 100947 rather than 34320/100947.
- 4) (i)** This was well answered. Some candidates lost a mark for failing to state that the sum of the probabilities is equal to 1.
- 4) (ii)** Again most candidates answered correctly but a number made things difficult for themselves by using decimals rather than fractions, often losing 1 or 2 marks due to inaccuracy. When calculating the variance some candidates subtracted  $E(X)$  rather than  $E(X)^2$ .
- 5) (i)** This was almost always answered correctly.
- 5) (ii)** Again most candidates gave the correct answer of 0.66 although a number had difficulty selecting the appropriate pairs, despite the fully labelled tree diagram being provided. A few candidates insisted on multiplying all their otherwise correct values together hence gaining only the first M1.
- 5) (iii)** Whilst approximately half of the candidature scored full marks, many gained just one mark for the correct denominator 0.66. It was far less frequent to see 0.54 as the numerator, 0.396 ( $0.6 \times 0.66$ ) or just 0.6 being two common errors. Some candidates calculated 0.54 and gave that as their answer, failing to recognise the conditional nature of the probability.
- 6) (i)(A)** This was almost always answered correctly.
- 6) (i)(B)** Whilst most candidates answered correctly, a number found the probability of exactly rather than at least two of the methods being used.
- 6) (ii)** Many candidates had the correct denominator but rather fewer had the correct numerator. The most common errors were 2/7, again not including the 4 where all 3 methods had been used, and 0.31 as the denominator showing candidates' lack of understanding of conditional probability.
- 6) (iii)** An answer of  $0.93^3$  was more common than the correct answer. A significant minority of candidates attempted to use a binomial expression, some of whom gained a method mark if  $0.93^3$  was the only value of significance (although it often was not). Several candidates gained from our acceptance of un-simplified fractional answers in this part.

- 7) (i)** Most candidates scored well on the histogram although a very small number plotted frequencies rather than frequency density. The usual errors of lack of label or incorrect label (frequency or even cumulative frequency when frequency density was being used) on the vertical axis, a non linear scale or inequality label on the horizontal axis and the occasional mis-plotting (10.9 instead of 11.9 for example) did occur but with less regularity than in the past. A small number of candidates omitted the working for the frequency densities.
- 7) (ii)** The estimation of the mean was answered well with the vast majority of candidates using the midpoints of the intervals and multiplying by the appropriate frequencies. A very small number used class boundaries or slightly incorrect midpoint values such as 10.5 instead of 10. A few replaced the midpoints by the interval widths, possibly because they had been used in the first part of the question. A relatively large number of candidates gave their answers to 5 or more significant figures, thus losing a mark for over-specification.
- 7) (iii)** Candidates were less successful in finding the standard deviation, due either to the use of an incorrect formula or to the omission of the frequencies. The use of  $\sum f(x - \bar{x})^2$  was seen but rarely did it produce an accurate answer. Few candidates seem to use their calculator functions to check the accuracy of their answer. Very few incorrectly found the root mean square deviation rather than the standard deviation. Some candidates lost a mark in this part due to over-specification.
- 7) (iv)** Most candidates knew that they needed to use the mean  $\pm 2$  standard deviations to establish the possibility of outliers although a few tried to use the quartile method or produced a written argument unsupported by any figures. A number lost a mark because they failed to specifically state that there almost certainly some outliers. A significant number of candidates wished wrongly to exclude the outliers because 'it was grouped data' or because 'we do not know how many there are' or because 'there are too many outliers as 107.26 is much less than 200'.
- 7) (v)** Very few candidates scored full marks in this part. Most found the new mean correctly but it was very often over-specified, even when the original mean in part (ii) was not. It was extremely rare to see the correct  $1.15^2 \times 33.25^2$  or  $(1.15 \times 33.25)^2$ . The most common wrong answer was  $1.15 \times 33.25$  (called the 'new standard deviation' or 'new variance' seemingly at random) followed closely by  $1.15 \times 33.25^2$ . Even the very few who wrote  $1.15^2 \times 33.25^2$  often spoilt their final answer by over-specification.
- 8) (i)** Nearly all candidates knew that the expected number of wins was calculated using  $np$ . However some rounded to 2 or occasionally 3, losing a mark in both cases.
- 8) (ii)(A)** This part was also well answered. Most candidates used the formula successfully, though occasionally the binomial coefficient was forgotten or the power of 0.8 was given as 8 rather than 10. The binomial coefficient was rarely omitted. A few candidates used tables, usually correctly.
- 8) (ii)(B)** Most candidates used tables for this part but a significant number chose to calculate the  $P(X=0)$  and  $P(X=1)$  and subtract. The majority of candidates arrived at the correct answer by one of these methods. There was some confusion between inequality statements and many used the wrong notation even if they selected the correct value from tables. Common wrong answers included  $1 - 0.2749 - 0.0687$ ,  $1 - P(X \leq 2) = 0.4417$  or just  $P(X \leq 2) = 0.5583$ .

**8) (iii)**

In this final part most candidates either scored three marks or fewer (for the hypotheses) or at least 7. The hypotheses, on the whole, were well defined. The vast majority correctly used  $p$  rather than some other letter, though  $X$  appeared occasionally and sometimes no letter was used at all. Once again the definition of  $p$  was often absent though more made an attempt than in the past - the mistake this year was to miss out the fact that it was the probability that **Ali** won the game. The reason for the choice of alternative hypothesis was not always clearly defined; sometimes it could have been mistaken for the hypothesis given in words rather than a separate explanation. Again confusion over use of inequalities was often seen, candidates writing  $P(X=7)$  even when they meant  $P(X \geq 7)$ . Point probabilities continue to be the preferred wrong method in this question, but they were seen less than in the past. Another common wrong method was  $1 - P(X \leq 7) = 0.0321$ , rather than  $P(X \geq 7)$ , leading to the opposite conclusion. Of those using the C.R. method, many also made this mistake, resulting in a C.R. of (7,8,9,... 20), again with the consequent opposite conclusion. Some candidates still failed to compare their probabilities with the significance level, though this was seen less than in previous series. Although it is given in the mark scheme, it is worth repeating here the recommended method for comparing the probabilities with the significance level: Candidates should find the two in this case upper tail cumulative probabilities which straddle the significance level and compare them both with the significance level.

$$P(X \geq 7) = 1 - P(X \leq 6) = 1 - 0.9133 = 0.0867 > 5\%$$

$$P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9679 = 0.0321 < 5\%$$

The decision whether or not the value was significant was usually correct for the candidates who got this far. Explanations were, on the whole, very pleasing. Most candidates indicated in some way that there was 'not enough evidence' though a few still fail to put their conclusion in context. However the answers to this question showed a marked improvement from those in the past. Thank you to the Centres for taking note of the comments made in previous sessions. It is worth reiterating here the point made in the General Comments above 'Some candidates did not state any probabilities to justify their critical region. In the past this omission has been treated generously, but in future candidates who fail to do so can expect to receive very few if any marks.'

## 4767 Statistics 2

### General Comments

Yet again, a very good overall standard of work was seen from the majority of candidates. There was no evidence that candidates struggled to complete the paper in the time allocated. Indeed, many candidates found time to write lengthy descriptions when asked for 'brief' comments. There were a small number of instances where candidates were penalised for providing excessive accuracy in final answers; more commonly, candidates were penalised for premature rounding which led to inaccuracy in final answers. Most candidates handled probability calculations competently. In general, the wording and layout of responses to hypothesis tests was very good.

### Comments on Individual Questions

- 1 (i) This was well answered, with many gaining full marks. Those candidates who used a version of the formula for calculating Pearson's product moment correlation coefficient,  $r$ , which involved calculation of the mean value of  $x$  and the mean value of  $y$  tended to round these values; this produced a large error in their final value for  $r$ . Attempts at both methods highlighted in the mark scheme were seen; occasionally candidates mixed these methods up finding, for example, the numerator from the  $S_{xy}$  method and the denominator from the covariance method.
- 1 (ii) In this hypothesis test, many candidates preferred to state their hypotheses in words, rather than using symbols. In both cases, many candidates failed to make it clear that they were testing for correlation in the underlying population. Most managed to carry out the required two-tailed test successfully. A small proportion of candidates did not manage to obtain the correct critical value, with some inappropriately using a one-tailed test value and others using critical values from the Spearman's table. It is a requirement in these tests that candidates show clearly a comparison of their test statistic with their critical value. This comparison can be in the form of a diagram; indeed, those who provided diagrams tended to be less likely to compare their negative test statistic with a positive critical value. It was pleasing to see most candidates phrase their conclusion along the lines of 'there is insufficient evidence to suggest that there is a correlation between ...' rather than the more assertive 'there is no correlation between...'.
- 1 (iii) The phrase 'bivariate Normal' seemed elusive to many candidates. The idea that if the points in the scatter diagram fall within an ellipse then the test is likely to be valid was more widely known. Some candidates referred to the diagram being 'linear' as the reason for validity.
- 1 (iv) Many found this difficult but some good responses were seen. Most picked up marks for recognising that the additional value was not within the original body of data and stating that the validity of the test was brought into question, despite providing spurious justifications. The mark for identifying the position of the point in the bottom right hand corner of the diagram was less frequently given. The most elusive mark proved to be the mark for commenting that the new value reduced the elliptical shape of the scatter diagram.
- 2 (i) The majority of candidates found it easy to justify the given value of the sample mean, but many struggled with the calculation of the sample variance; calculation of MSD was common. This led to difficulties providing a sensible answer to part (ii)

- 2 (ii)** This was well answered by those obtaining the sample variance in part (i). It was clear that some candidates knew what was required in part (ii) but did not know how to calculate sample variance – many 'fudged' their answers to part (i) to enable them to answer part (ii).
- 2 (iii)** This was very well answered, with many candidates gaining full marks. Most commonly, marks were lost in part (iii) *B* for calculating  $1 - P(X = 1)$ , or for rounding  $\lambda$  to 1.8 so that tables could be used.
- 2 (iv)** Very well answered, with most candidates managing to satisfactorily justify the given answer.
- 2 (v)** Also well answered. Some candidates proceeded to use their own (wrong) answer from part (iv) despite the correct value being provided.
- 2 (vi)** Many candidates struggled with this request to use a suitable approximating distribution to find the required probability. Most decided to use a Normal approximation, but seemed unsure as to the correct parameters; in particular the variance caused problems. In addition to this, many did not use a continuity correction or used 29.5 instead of 30.5. Otherwise, the majority of candidates coped well with the probability calculation, demonstrating a good understanding of how to use the Normal distribution.
- 3 (i)** Very well answered. Many candidates gained full marks here. Where marks were lost it was usually through inaccurate use of Normal probability tables. Most candidates managed to standardise correctly; although some applied spurious continuity corrections and were thus penalised. A few candidates struggled with the structure of the probability calculation, particularly in part (i) *B*, but efforts at this were, on the whole, good.
- 3 (ii)** Well answered. Most candidates managed to identify the appropriate z-value, -0.8416, and use it appropriately to produce the correct answer. Common errors included using +0.8416 in place of -0.8416. Many others used a mix of probability and z-value by using  $1 - 0.8416$  (= 0.1584) in place of -0.8416.
- 3 (iii)** This was reasonably well answered, but proved to be one of the more difficult parts of the paper. Most candidates managed to state correct hypotheses, with only a small proportion opting to apply a one-tailed test. As usual, the definition of  $\mu$  as the population mean was omitted in most cases. A variety of approaches to the test were seen, with the method outlined in the main body of the mark scheme being most popular. Candidates who treated the **mean** value of 344 as a single observation from the  $N(355, 52^2)$  distribution were heavily penalised. Several candidates identified the critical value incorrectly as -1.645 despite carrying out a two-tailed test at the 5% level of significance. Again, it was pleasing to see most candidates phrasing their conclusion correctly as 'there is insufficient evidence to suggest that women have a different mean reaction time to men'.
- 4 (i)** Very well answered. This part proved to be a valuable source of marks for most candidates. Common reasons for marks to be lost included inaccurate recording of expected frequencies or premature rounding to, say, 1 decimal place leading to inaccuracy in the calculation of the test statistic. Otherwise the remainder of the test was handled well. A few candidates attempted a two-tailed test and were thus penalised. Very few candidates were penalised for not referring to the context of the question in hypotheses and/or conclusion. Very few mixed up association with correlation.

- 4 (ii)** As with previous years, candidates found this type of question difficult. The aim of the question was to get candidates to explain the connection between size of contribution to the test statistic and degree of association. It was apparent that candidates had a better idea of how to proceed than in previous years, with many referring to the particularly large and small contributions and making sensible comments. However, many candidates' comments did not refer to contributions either explicitly or implicitly; in such cases, full marks were not available. Those candidates who insisted on providing an explanation for each of the three sizes of pebble at each of the three sites often slipped up somewhere along the way and lost marks; if candidates answered the question in the desired manner, it was not necessary to comment on each of the nine contributions to obtain full marks.

## 4768 Statistics 3

### General Comments

There were 274 candidates from 38 centres (compared with January 2010: 280 from 41) for this sitting of the paper. Although there were several very competent scripts there was much work that was quite disappointing and poorly set out. Many candidates were unable to carry out basic tasks, not at all in keeping with what one might expect at this level.

Invariably all four questions were attempted. Marks for Question 1 were found to be higher on average than the other 3 questions. Question 4 seemed rushed at the end suggesting some candidates may have found themselves short of time.

### Comments on Individual Questions

- 1) (i) Although intended as a gentle start to the paper, it was surprising how many failed to score full marks here. Two faults prevailed: the standardised value 1.166...became 1.666... either when transferred from the calculator or when looked up in the Normal tables.
- (ii) There was more success here. Provided candidates were careful with the variance of the monthly charge then the correct result would usually follow. However some candidates did get caught out when trying to convert pence-squared to pounds-squared.
- (iii) There were more problems in this part when trying to sort out the variance, occasionally made worse by premature approximation.
- (iv) There were many good solutions to this part. However, it was often the case that marks were lost as a result of hypotheses that were imprecisely specified and/or conclusions that were inadequate. Many neglected to express the final conclusion non-assertively, in context and including wording such as "on average" to refer to the mean. Quite a few candidates based their test on the differences from 432 kWh, the old mean – a strategy that works but introduces additional opportunities for making mistakes.
- 2) (a)(i) The syllabus topic "Sampling methods" remains consistently and conspicuously badly understood by candidates. The definition and subsequent discussion of stratified sampling was usually vague and woolly. Few explained coherently and concisely the idea of a population that divides up into identifiable subgroups. Any reference to the strata being sampled randomly was often omitted. So also was the phrase "representative sample" as a reason for the use of this method. It should be noted that a stratified sample does *not* need to be selected in proportion to the sizes of the strata. There may be very good reasons for not doing so, for example if some strata are much more variable than others (it would be sensible to take more observations in the more variable strata), or if some strata are much more expensive to sample than others (it may be necessary for budgetary reasons to restrict the sampling in the more expensive strata).
- (ii) This part was well answered, but perhaps not as well as expected.
- (b)(i) Given that the Wilcoxon test usually provides one of the more successful questions from the point of view of candidates, it was disappointing to discover how few could explain the circumstances under which it would be valid to use this test. A very common wrong answer was that the data (sic) should be Normal.

- (ii) In most cases the calculation of the test statistic and the identification of the critical value were correct. Sometimes errors in the ranking arose through candidates misreading their table of differences. As in Question 1 (iv), solutions were let down badly by incorrect hypotheses and/or inadequate conclusions. Some candidates stated their hypotheses in terms of “the difference of the medians” which is not necessarily the same as “the median of the differences” and which should be discouraged.
- 3) (i) Many candidates were not able to find the sample mean and standard deviation from grouped data. A large number of them had little, if any, idea about how to set out the calculations.
- (ii) This was also badly answered. Many faked the answer by ignoring the instruction to use the Normal distribution, choosing instead to work out “100 – the sum of the given frequencies”. Furthermore, among those who did what was intended, a wrong answer was likely to be taken forward without checking its feasibility.
- (iii) The Chi-squared test was not carried out with the same competence as in the past. Errors abounded, notably the failure to merge classes at one end, at least, and the incorrect identification of the number of degrees of freedom resulting in an incorrect critical value. As in Questions 2 and 3 the hypotheses and conclusions were often expressed badly. A further common fault, mentioned in previous reports, involves statements such as “the data fits/follows the model.”
- (iv) The discussion of the outcome of the test rarely showed more than a superficial appreciation of what was going on. It was not uncommon for the wrong class to be identified as providing the largest contribution to the test statistic.
- 4) (i) The majority of sketches were considered to be adequate. One would like to think that students at this level could be relied on to label their axes “x” and “y”.
- (ii) Only a minority of candidates used the symmetry argument to “write down” the mean. As often as not, those who integrated got it wrong, largely because they could not apply the limits 0 and  $-1$  correctly in the integral for the left hand portion. The same problem with limits occurred with the variance, which frequently turned out to be 0, or even, on occasion, negative.
- (iii) Almost all named the required distribution correctly as Normal, and most gave the correct mean. Fewer candidates than usual could write down the correct variance and hardly any appeared to be aware of the Central Limit Theorem as the justification.
- (iv) Most of the time there was evidence of some understanding of how to construct a confidence interval using the sample mean and the correct Normal percentage point. What was worrying was that many candidates seemed unable to make the connection between  $\text{Var}(\bar{L})$  in the previous part and the standard deviation needed here: either a spurious extra  $1/\sqrt{50}$  was introduced or they ignored  $\text{Var}(\bar{L})$  completely.
- (v) Most candidates appreciated that an appropriate response to this part depended on whether or not 90 was contained in the interval in part (iv).

# 4771 Decision Mathematics 1

## General Comments

Candidates found this paper difficult. All questions contained routine applications, but all questions also contained elements which challenged candidates to think and to consider what one would expect really in Decision Mathematics.

Of course, it is always very difficult to think clearly. It is the skill which marks out good mathematicians. Those not so good at it were particularly caught out when explanations were required, as in parts 1(iv), 3(iii), 3(iv), 5(vi), 6(i) and 6(ii) however, literacy and mathematics go hand-in-glove, both depending on that elusive clarity of thought.

## Comments on Individual Questions

**Q 1** This question explored the relationship between electrical circuits and corresponding connectivity graphs. Many candidates did not absorb or understand the phrase “either directly or indirectly”, which was crucial to the question. This led to many errors in parts (i) and (ii).

Part (iii) was entirely graphical, and it was extremely disappointing to find the majority of candidates splitting the graph into sets of two and three vertices by deleting 6 arcs. Some even pondered on deleting the 4 arcs attached to a vertex and decided against doing so since they could not conceive of a single vertex as constituting a set.

Successful answers to part (iv) required reference either to the distinction between direct and indirect connectivity, or to the distinction between arcs and wires. This is representative of the underlying ethos of Decision Maths. We are modelling, and we need always to be able to distinguish between the model and reality. Candidates should be alert to the language which indicates this distinction, in this case “wires” and “arcs”.

Mention must be made of the significant number of candidates in part (iv) who tried to install switches, arguing that opening switches did not constitute cutting wires.

**Q 2** This question focussed on testing the ability of candidates to follow instructions. The majority were found wanting, their solutions incurring unnecessary tests and costs.

**Q 3** Part (i) was entirely routine, and potentially well-rewarded. It was very disappointing that so many candidates fared so badly with it. For instance, many candidates, inexplicably, labelled vertex E correctly, but then moved straight on to F, forgetting to consider D. Even more candidates neglected altogether to answer the question, or only answered one part of it, route or distance. Echoing comments above, many seemed unable to distinguish between “route” and “distance”.

Part (ii) saw the weakest set of answers to any part of the paper. Very few correct explanations were seen. Popular misconceptions included “To find the shortest route to F we have to find shortest route to all other vertices” and “The route to F visits all other vertices” ... it doesn't! Given the difficulty which many candidates had in expressing themselves, it was often difficult to distinguish between those who thought that all other vertices were always labelled by Dijkstra, and those who correctly observed that on this occasion all other vertices happened to have been labelled. For instance, “It is necessary to visit all other vertices to find the shortest route to F”.

Rather more candidates were successful in gaining the mark in part (iii), some very succinctly, e.g. “B is in the shortest path from A to F”. Some candidates were keen to point out that B was the second vertex labelled (true), and is therefore in all shortest

paths (not true). It was sometimes difficult to distinguish between that answer and answers pointing out that B is the second vertex on the shortest path from A to F, which of course did earn the mark.

- Q 4** Most candidates showed some ability to identify precedences, although as ever, “immediate” predecessors created some difficulties. Parts (ii), (iii) and (iv) were routine, but still caused some candidates problems. Only about half were able to produce a precedence diagram which was correct in all respects, most failing with aspects of H, I and J. Surprisingly often, arithmetic errors were made on the backward pass, particularly with 6.5-2 and with 6.5-1.5. The published markscheme shows a compressed version of a cascade chart for part (v). This is acceptable, since some find it easier to produce. Many candidates' efforts were virtually illegible after scanning ... candidates really **MUST** ensure that their work is clear. Some candidates omitted activities B, D and F, which was not acceptable ... they did not require resource, but they were needed for the precedence structure. Many solutions did not focus first on the critical path, started E late, and failed to finish in 7 minutes. Candidates did not seem to notice this.
- Q 5** Most candidates were able to produce acceptable rules in parts (i), (ii) and (iv), although every anticipated error was seen. Also as anticipated, about half of the candidates failed to answer the question in parts (iii) and (v), ignoring the injunction “showing which of your simulated chances are converted into goals”. Many of those who did show which, and many who did not (there was evidence), failed to do the simulation correctly. They used all of the provided random numbers instead of the number needed, i.e. simulating 9 chances in each case instead of the numbers of chances determined earlier. Part (vi) provided a stark example of the criticism voiced above ... lack of clarity. One could imagine most of the candidates being able to debate the relative merits of creating many goal scoring chances but accepting few, against creating fewer chances but being better able to convert them. But they were required to compare goals scored ... that is what the simulation was about! In part (vii) candidates who talked about “using more random numbers” were not making themselves clear. More repetitions (correct answer) require more random numbers, but so, in a different sense, does using 3-digit random numbers. The latter is incorrect since it does not answer the question ... it would affect the efficiency but not the reliability.
- Q 6** There were many inadequate answers seen for parts (i) and (ii). The examiners require just short and clear comments. Paradoxically, candidates were more at home with the  $5a + 6b \geq 27$  than with other parts, and some good algebraic reasoning was seen there, as per in the markscheme, i.e.  $5(a+2) + 6(b+4) \geq 61$ . Some good verbal explanations were also seen, and were equally acceptable. Quite a large number of good answers were seen to 6(iii). The most common, and distressing error, was to see scales marked and used on  $b = -4$  instead of on  $b = 0$  (the  $a$ -axis), and on  $a = -2$  instead of on  $a = 0$  (the  $b$ -axis). This revealed a misunderstanding of the most fundamental of mathematical concepts involving the use of graphs. Some candidates were able, by hook or by crook, to recover inadequate earlier work in parts (iv) and (v), and this was allowed.

## 4776 Numerical Methods (Written Examination)

### General Comments

There was a lot of good work in response to this paper. The standard methods were well understood and there was little evidence of unprepared candidates being entered. As ever, the layout of work was not as good as it could be. Candidates should realise that, in numerical methods in particular, laying work out systematically and compactly aids accuracy. Figures scattered wildly around the page are far less likely to be correct.

Interpretation and real world application (as required in Question 4) remain areas of relative weakness.

### Comments on Individual Questions

- 1) Parts (i) and (ii) were done well by almost everyone. The vast majority moved immediately to the 'inverse' iteration in part (iii) and finished the question successfully. Some used other methods (bisection, secant, Newton-Raphson). These were given credit where successful, though they generally involve more work.
- 2) Most candidates appreciated what to do here, though some did not realise that they could calculate *two* further trapezium rule estimates. A small minority of candidates obtained a further mid-point rule estimate by extrapolation; this was not required, and its lack of accuracy makes it dubious, but it was not penalised.
- 3) This was a very straightforward question and almost everyone knew exactly what to do. The comments on the relative merits of the estimates were mostly correct.
- 4) There were a lot errors made in answering this question. Some were very basic: errors with units (pounds and pence) or with days and years. Others were more fundamental: chopping to the nearest 0.01 was often said to produce a maximum possible error of 0.009. And quite a few candidates were evidently unsure of the difference between chopping and rounding.
- 5) This was another very straightforward question. The only notable errors were arithmetical – usually arising when subtracting one negative number from another.
- 6) Part (i) was generally done well, though candidates' layout of their answers made for very difficult reading in many cases.

In part (ii) the justification for the new approximation is twofold: the errors in  $g(x)$  and  $h(x)$  are in the ratio 1:4, *and* they are of opposite sign. It was rare for candidates to make both points.

Part (ii) was often poorly answered even though it is based on a commonly examined topic, the subtraction of nearly equal quantities.

7) In part (i) the root was easily located, but the arguments to show that there are no other roots were mostly poor. Very often candidates set the first derivative to zero, identified that  $x = 0$  was the only solution, and then said that the function had only one turning point and so it could have only one root. It is difficult to see what 'reasoning' lies behind that. It was refreshing to come across a script in which the candidate observed that the derivative was never negative and then made the correct deduction.

In part (ii) the numerical work was done well, but the sketches were sometimes disappointing. In quite a few cases it appeared that candidates knew what the sketch *should* show but didn't draw a graph large enough to show it. A small minority appeared not to understand that the Newton-Raphson method approximates a curve by its tangent and tried to draw staircase or cobweb diagrams.

Part (iii) was done well in most cases, though some thought that the first iteration was diverging.

## Coursework

### General Administration

Centres are reminded that the deadline date for the submission of marks to OCR is December 10. This is to ensure that a sample request can be generated which will give centres the chance to receive it and despatch the sample to the moderator before breaking up for the Christmas break. Most centres complied most helpfully but a number did not, resulting in the receipt of coursework by the moderator well into January. A few centres sent all the work in good time to the moderator but failed to submit their marks to OCR. Without knowledge of the sample determined by OCR, moderators are unable to proceed which causes the same problem as above.

For a handful of centres there was a communication problem in that they asserted that the sample request had not been received. This may be because the email address held by the board is not correct. For centres using a general email address for the examination officer, this will not happen, but when an address uses the personal name of the examination officer then a change of personnel will render the email address invalid. Centres are asked to ensure that the details of the centre held by the board are correct.

Centres are required to fill in an authentication form (CCS160) and this should be sent with the work to the moderator. Failure to do so results in extra communication which takes time.

Centres are requested to use the comments boxes on the cover sheet and to annotate the work where it has been checked and to give criterion marks rather than domain marks. We have seen instances of tasks which have had no comments and no annotations whatsoever. Likewise we have seen instances where the work has been clearly incorrect but ticked as correct.

Centres are reminded that it is a requirement to supply a brief report on the Oral Communication domain.

### 4753/02 – Methods for Advanced Mathematics, C3

It is of concern that the marks of rather more centres have been adjusted this series. The reason is because credit is being given for work that does not satisfactorily meet the criteria. There are no new points to be made and so the implication is that assessors are not noting what is being said in these reports. Centres are urged to use reports on coursework to inform their assessment.

We will outline the difficulties for each of the criteria in this specification.

### Change of sign

- Most candidates do a decimal search. The root should be stated (rather than a range being given) and it should be correct to at least 3 decimal places. A number of candidates took, for instance, the range [1.11, 1.12] and asserted that the root was 1.115 correct to 3 decimal places.
- A graph of the function does not constitute an illustration.
- The following equations should not be used to demonstrate failure: trivial equations, equations with no roots, equations with a root that is found in the table.

### Newton-Raphson method

- The roots should be found to at least 5 significant figures. We expect to see the working for at least one root which demonstrates an understanding of the method. This means seeing the formula developed from the general Newton-Raphson formula for the particular equation (including sight of the derived function). Screenshots of "Autograph" may be used for subsequent roots but does not in itself demonstrate an understanding of the method.
- If an equation is used which has only one root the second mark should not be awarded.
- As with the previous method, a graph of the function does not constitute an illustration. We expect to see two clear tangents which match the iterates.
- Error bounds need to be established, typically by change of sign, rather than simply stated.
- This method can be shown to fail if an initial value close to one root actually converges to another. Typically, an initial value that is "close to the root" may be an integer either side of the root. Taking an initial value that is not close enough to "demonstrate" failure to converge to a stated root is not acceptable. Likewise, we do not expect a "contrived" initial value just because it happens to be a turning point.

### Rearrangement method

- Although there is no stipulation for error bounds in this criterion, nor is there any demand for a specific accuracy, it is expected that candidates will give a specific value for the root, and be aware of the accuracy of their root.
- A graphical illustration will show either a staircase or cobweb diagram. This diagram should match the iterates found. The magnitude of  $g'(x)$  can be discussed in two ways. The gradient function,  $g'(x)$  can be found and calculated for a value of  $x$  that is close to the root and referred to the criterion for convergence. (The initial value of  $x$  is not usually close enough.) Alternatively, the gradient of the curve  $y = g(x)$  can be discussed in general terms in relation to the way in which the curve cuts the line  $y = x$ .
- The same equation should be used to demonstrate failure. The same rearrangement may be used to attempt to find another root, or a different rearrangement may be used to find the same or another root.
- As with the success, a clear diagram should be drawn to demonstrate divergence using the iterates found and the value of  $g'(x)$  discussed.

### Comparison

- When making a comparison of the fixed point methods, the same initial value should be used to find the same root to the same degree of accuracy.
- Without this, the discussion of speed of convergence is not valid. It is expected that the number of iterates required in each method to find the root should form part of this discussion.
- Candidates should refer to the hardware and software available to them in working this task. Different candidates will have different resources and will come to different conclusions.

### Terminology

- This domain was added in the last revision of the specification to give assessors the opportunity to penalise candidates who do not use the correct terminology. Many assessors do not take this opportunity and give the full mark regardless of the terminology used. Typical errors which should be penalised are: failure to write equations (referring, for instance to  $y = f(x)$  as an equation to be solved), incorrect language (for instance "I am going to find the root of the graph" ) and candidates who word-process their reports but are unable to write subscripts and powers properly.

### 4758/02 – Differential Equations

Only a relatively small number of centres submitted work. Work submitted was of a high standard and, in the main, assessed well. As a result, the proportion of adjustments made was lower than usual. In spite of this, there are certain points worth making.

- It would seem to make sense to give and state the source of the data at the beginning of the work, in order to introduce the problem, in spite of the fact that the criteria for this is given in domain 3.
- The data themselves, ideally in graph and table form, should be presented and there should be some discussion regarding their accuracy.
- Assumptions are a vital part of the modelling process, both initially and when amending the model. By and large the initial assumptions are generally satisfactory, but often when it comes to amending the model by considering a revision of the initial assumptions, investigations can veer towards curve fitting unless there is some justification for the revision.
- Whilst it is not always possible, particularly with the usual modelling exercises, to avoid using the data to predict the data, this should be avoided when using the modelling / experimental cycle. Here it is advisable to use the results from one experiment to derive parameters which can then be used to predict the outcomes of a subsequent experiment or experiments.
- There are occasions when the modelling assumptions used initially result in a Differential Equation that is too simple for this level of work. Coursework is to do with work within the course!
- It is also expected that the initial model developed will be worked for the whole motion and not just part of it. This is particularly so in the task "Aeroplane Landing"

Finally it is apparent, in some cases, that the coursework has been conducted during the early stages of teaching this module. Therefore some techniques which are necessary for the complete solution of an investigation are not known and inappropriate methods are used. Although one is sympathetic with this issue, none the less, the script should be marked down accordingly.

### 4776/02 – Numerical Methods

Most candidates attempted suitable tasks, but in a small number of cases a heavy penalty was incurred by candidates doing lightweight tasks and only nominally meeting the assessment criteria. Where this penalty was not imposed by the assessor then some large adjustments had to be made. The following points might be useful for teachers and assessors to avoid this problem in future submissions.

- Not all candidates are able to give a correct formal statement of the problem. This is seldom penalised by assessors.
- Many candidates simply describe the methods they intend to use, rather than justify their selection, for the second mark in the second domain.
- In numerical integration we have reported before that we consider a "substantial" application to be one where the investigation extends to at least 64 strips. Some candidates are still given full credit for only going as far as 16 strips.
- Most do not deserve the second mark in domain 4, yet it is frequently awarded. Often there is simply a description of what software was used.
- Too many candidates compare their values with known values –  $\delta$  or values obtained from the MATH function on a graphical calculator. A few use the theoretical values for  $r$  even when there is compelling evidence that this is inappropriate, and some candidates still extrapolate from (say)  $S_4$  and obtain a less accurate approximation than their final solution.
- Few candidates are able to argue coherently for a stated level of accuracy referring only to their iterates. Limitations were often simply ignored, but given full credit.

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