OCR
Oxford Cambridge and RSA

GCE

Mathematics

Advanced GCE A2 **7890 – 2**

Advanced Subsidiary GCE AS **3890 – 2**

OCR Report to Centres June 2016

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OCR REPORT TO CENTRES

4721 Core Mathematics 1

General Comments

As usual, the vast majority of candidates were very well prepared for this paper and even though the level of demand was slightly higher than in some recent sessions, many candidates achieved very high marks. The use of additional sheets remains uncommon and many of those that did need to repeat a solution indicated so clearly, which was very helpful to markers. A few, however, still leave a choice of answers which should be discouraged. Any "rough work" should also be clearly indicated as such. It should also be noted that as scripts are scanned for marking purpose, schools should advise candidates to write clearly in black so that solutions can be more easily read.

Many candidates presented clear, well-structured and accurate solutions throughout the paper, although some provided unsupported answers in the later stages of the paper which were difficult to award credit. In particular, the solution of quadratics – disguised or otherwise – remains very strong with factorisation much more common than attempts at the quadratic formula. Circle and co-ordinate geometry remain strong, as does basic differentiation. A small number of challenging questions caused difficulty for some candidates. The general standard of response to question 8 indicated a lack of awareness of differentiation form first principles. The degree of algebraic manipulation needed for question 9 proved beyond some candidates' reach. Transformation of graphs continues to prove challenging.

It was noticeable that many solutions that showed procedural understanding were let down by poor arithmetic including sign errors, failure to simplify integer answers and difficulty with fractions; many marks were lost because of this, notably in questions 3, 4 and 10.

Comments on Individual Questions

1) (i)   Almost all the candidates secured the first mark for expanding one of the quadratic expressions correctly; the vast majority also simplified accurately. There were, however, a number of errors in dealing with the negative coefficients of the second expression so that only about three-quarters of candidates secured both marks. Some went on to "solve", which for this starter question was ignored.

(ii)   Although some candidates chose to multiply out the given expressions fully, many candidates successfully identified just the terms necessary to give x^3 and saved themselves a lot of effort. This was largely done accurately, more successfully than the first part, and although sign errors occurred both in the individual terms and in combining with e.g. -6 – 4 = 10 fairly regularly seen.

2)   Most candidates recognised the need to rationalise the denominator and did so efficiently and accurately, with many candidates securing all four marks. The conversion from $\sqrt{20}$ to $2\sqrt{5}$ was usually well done; most errors that occurred were seen when expanding and simplifying the numerator. Some candidates obtained the correct answer but then, seemingly unsatisfied with the fractional values of *a* and *b* found, multiplied by 4; this lost the final mark.

3)   This familiar question was very well done with many candidates scoring full marks. The vast majority of candidates opted to substitute for *y* and so form a quadratic in *x*. There were some errors, for example 16 – 34 = 22, but most substitutions were very good and clearly shown. As in most recent sessions, candidates remain more likely to factorise, accurately, rather than depend on the quadratic formula. This usually resulted in the

correct values of *x*, but a significant number of accuracy errors then occurred when substituting for *y.* Forgetting to work out the second variable was not entirely absent.

4) This disguised quadratic was well approached by the vast majority of candidates, with around three-quarters of candidates achieving all 5 marks. A very small number of candidates factorised into two brackets, but the most common approach was as usual to perform a substitution and then to factorise. There was some confusion with choice of substitution with many incorrectly obtaining $2x - 7x^2 + 3$; this earned no credit. Again, very few candidates used the quadratic formula and factorisation was usually successful with only a few sign errors seen. Some candidates stopped after solving the quadratic and a small number tried to take the fourth root, rather than to raise to the power four. Some only squared, implying an incorrect substitution. A few gave answers like ±81, which lost the final accuracy mark, as did poor attempts at $\left(\frac{1}{2}\right)^4$ which was variously seen as $\frac{1}{32}$, $\frac{1}{64}$ or $\frac{1}{256}$.

5)(i) This simple index question was very well done, with around 90% securing both marks.

(ii) Although there were a significant number of excellent solutions, this question proved much more demanding than expected with less than a third of candidates securing all three marks. Many reached $5 \times 2^{\frac{4}{3}} + 3 \times 2^{\frac{4}{3}}$ but then went no further, or even "simplified" this to $10^{\frac{4}{3}} + 6^{\frac{4}{3}}$. Many of those who did obtain $8 \times 2^{\frac{4}{3}}$ appeared not to realise 8 was a power or 2. Some of those who did then made errors adding the powers, either through incorrect addition or multiplying so that $5 \times 2^3 \times 2^{\frac{4}{3}} = 2^4$.

6)(i) The negative coefficient of x^2 generally did not daunt candidates and there were many clear and accurate solutions, aided by the integer arithmetic. There were the usual errors when trying to find the constant term and these were exacerbated by the need to multiply two negative numbers together. Some candidates however, chose to change all the signs to make the question easier; this approach earned a maximum of one mark in this part, with the possibility of follow through marks in part (i). Others treated the expression as an equation to achieve the same effect; at this level it is expected that candidates should know the difference.

(ii) This part was sometimes omitted with candidates apparently not seeing connection between the parts. Others found the coordinates by differentiation and substitution but most used (i) and so were allowed follow-through marks had they made errors in the previous part.

7)(i) This graph sketching question caused a number of difficulties. Although most candidates recognised that this was a cubic, not all realised it was negative. The double root at $(0, 0)$ was often missed, with some transferring this to $(3, 0)$ to make their graph "work". Some graphs were carelessly drawn with implied extra roots with candidates apparently not considering the behaviour of the function at extreme values. Those who drew quadratics earned no marks.

(ii) This proved to be a very challenging question. Even those who found a correct equation did not simplify, or simplified incorrectly. The most common error was to only consider one term leading to, for example, $y = (x-2)^2(3-x)$. Sometimes a different translation was applied to each term e.g. $y = (x-2)^2(1-x)$. It was relatively rare to see the translation mistakenly performed vertically.

(iii)   The use of correct mathematical language to describe transformations continues to improve, but some instances of phrases like "in the *y*-axis" are still seen. The correct word "stretch" was the most commonly seen, although some did use "enlargement". A common error was to describe the stretch as "scale factor 2 parallel to the *x*-axis".

8) (i)   Less than two-thirds of candidates secured all three marks for this part, and although some used the fact the answer was given to go back and correct slips in algebraic working, others obtained answers that were clearly "fiddled". Many attempted to use differentiation rather than the correct method to find the gradient of a line segment.

(ii)   Correct answers to this were rarely seen. An appreciation of the understanding of a limit was expected, but many just used differentiation to compare the values or, even more commonly, discussed the "negative reciprocal".

(iii)   Most candidates were able to access this question, although some still worked with the line segment rather than the point *A* and so were unable to earn credit. The arithmetical demand led to the loss of accuracy marks in many cases.

9)   Although this question proved demanding to many candidates, there were a large number of neat solutions. Most candidates understood the nature of the question and many gained at least three of the four method marks available. Accuracy was the main barrier to complete success as a large number either failed to rearrange the given equation correctly or to substitute correctly into the discriminant. Repeated sign errors often resulted in apparently correct critical values for *k* that did not receive credit as they were from wrong working.

10)(i) This proved to be the most successfully answered question on the paper, with around nine in ten candidates securing all three marks.

(ii)   Just over half of candidates obtained full marks in this part, with errors appearing at all stages. Some put *x* rather than *y* equal to 0 when trying to find A and the alternative method of using Pythagoras' theorem often led to slips. There were a significant number of problems finding the gradient and errors such as $-\frac{3}{6} = -\frac{1}{3}$ were commonly seen.

(iii)   Many candidates did not realise that the point required for the parallel line was the opposite end of the diameter. Most did use the same gradient as in (ii), but some used the negative reciprocal. An interesting method sometimes seen was consideration of translation of the original line.

(iv)   This proved very demanding, with many candidates unable to start; those who drew a diagram were generally more successful but less than a quarter of candidates secured both marks. Even amongst those who found the maximum length of the radius to be $\sqrt{45} - 5$, it was quite rare to see the correct inequality.

11) Many candidates obtained at least the first four marks for this demanding final question, by correctly differentiating and setting equal to zero; the most common errors at this stage were to equate to 32 or to leave the constant term 5 in the derivative. Thereafter, a significant proportion candidates went on to secure at least 7 of the 8 marks by finding an expression for *a* and correctly substituting this and 32 into the equation of the curve, or other equivalent methods. Some did not spot this way forward and others lost marks due to incorrect simplification of algebra or arithmetical slips. Many did not spot the factor of 3 in $x^2 = \dfrac{27}{12}$ and so were then unable to finish the question. Occasionally candidates appear to consider *a* to be a variable passing through the point (*x*, 32); often these attempts were unclear and involved the (often unrealised) creation of functions of the form *xy* and the subsequent attempts at implicit differentiation were incorrect.

4722 Core Mathematics 2

General Comments:

The vast majority of candidates were well prepared for this examination, and able to make an attempt at every question within the allocated time. There were many well presented scripts, and in general candidates are showing more detail of the methods used than seen in previous sessions. The use of technology is to be encouraged, but candidates must be aware that simply writing down values is unlikely to be sufficiently convincing to gain any method marks. This is particularly pertinent when solving equations and when evaluating a definite integral.

In questions where the answer is given, candidates should ensure that they show sufficient detail so as to be fully convincing in their method. They must also take care in the detail shown as any wrong working will be penalised, even if it results in the expected answer.

Candidates should also ensure that they make effective use of brackets, both in order to convince the examiner that the correct method is being used but also to ensure that candidates do not subsequently misinterpret their own working.

There were a number of questions where candidates needed to be familiar with mathematical terminology such as 'coefficient', 'constant term', quotient', 'roots' and 'period'. In a number of solutions it was apparent that candidates were not familiar with these terms which had a detrimental effect on their ability to fully and correctly answer the question posed. Candidates should also pay due attention to any instructions in a question with regard to the required format of the final answer. This includes requiring the answer to be given exactly, to be given in a fully simplified form or to be given to a specified degree of accuracy.

Comments on Individual Questions:

1(i)   This was a straightforward start to the paper, and nearly all of the candidates were able to find the correct value for the length. The most common and efficient approach was to use the sine rule, but other methods were also employed. As ever, a few candidates worked with their calculator in radian mode, and persisted with their solution despite it resulting in a negative length. As always, candidates should check the reasonableness of their answer and review their method if necessary.

(ii)   This part of the question was also very well answered by the majority of candidates, and full marks were very common. The cosine rule was usually quoted correctly, but candidates who are unsure should make use of the formula book. Some candidates were unable to correctly evaluate the expression with additional, incorrect, brackets being used or square rooting being omitted.

2(i)   Candidates appreciated how to convert from degrees to radians and the vast majority were able to make a good attempt at doing so. The question requested the angle in an exact, simplified, form, and most candidates complied with this request. Some candidates spoiled an otherwise correct answer by subsequently giving a decimal approximation, and this was penalised.

(ii)   Many candidates completed this part without difficulty, providing concise and elegant solutions, but it was found to be challenging by others. Candidates were able to recall the expression for the arc length, but some did not incorporate this into an expression for the entire perimeter and others used the angle in degrees rather than radians. Once the correct equation for the perimeter had been found, the manipulation of algebra required to

solve it proved to be challenging. Rather than combining $2r$ and $0.3\pi r$ by either using a common factor of r or using a decimal approximation for 0.3π, the more common approach was to divide 60 by 0.3π and equate this to $2r$. Correct algebraic processes had to be employed to gain more than one mark on this question.

3(i)   This part of the question was very well answered, and the majority of candidates gained all of the marks available. Most candidates used the binomial expansion and made efficient use of brackets in obtaining a fully correct solution. The most common error was to either not use brackets at all, or to ignore the brackets that had been used earlier, resulting in an expansion where the powers of k were incorrect. A few candidates expanded the three brackets, and this was usually also done correctly.

(ii)   By contrast, this part of the question proved to be challenging for all but the most able candidates. Most were able to correctly identify the coefficient of x^2 but were unsure as to the 'constant term', with 3 and k being the most common errors. The other common error was to equate entire terms, rather than just coefficients, resulting in x^2 only being present on one side of the equation. Both of these misconceptions demonstrate the importance of candidates being aware of the correct terminology as well as the correct processes.

4(i)   The majority of candidates were able to produce a fully correct solution to this part of the question. Of the remainder, most were aware of the power law but too often this was not used as the first step or the second term was incorrect at this stage so no fully correct expression was ever seen. Some candidates obtained the correct expression but then incorrectly cancelled within the logarithm, which was penalised. Another relatively common error was for the difference of the two logarithms to result in a fraction with a logarithm appearing in the denominator. Even if this subsequently was written as the required single term, the error in the method was still penalised.

(ii)   Most candidates who had correctly combined logarithms in the first part of the question could then carry out the correct process to remove the logarithms in this part of the question and solve the ensuing equation with ease. Only the most astute candidates appreciated that -3 was not a valid solution to the given equation and thus needed discarding, which meant that three out of four was the modal mark. To gain any credit in this part of the question it was expected that there had been a valid attempt in part (i) to write the two logarithms as a single term.

5(a)   All candidates appreciated the need to expand the brackets as the initial step, and this was done correctly by most. The integration attempt was mostly correct, with a few candidates omitting the constant of integration. Even when the initial expansion was incorrect, the majority of these candidates could then integrate their function correctly.

(b)(i)   The function being given in index form made this question accessible to most students, who were able to gain the first two marks for a correct integration. Many candidates were then able to attempt to use limits correctly, but in some cases the lack of detail shown meant that there was no clear evidence of a correct method being attempted. A lack of fluency in manipulating indices meant that a significant minority were unable to gain the final mark for a correct simplified integral. A number of candidates then either equated the expression to 0 and attempted to solve the equation or used an inequality to link it to 1.

(ii)   The majority of candidates gained this mark, with the more astute candidates deducing the value from their integral in terms of a. A number of candidates did not fully appreciate the connection with the previous part of the question and instead restarted from the substitution of limits, often repeating any errors made in the previous part. There was considerable confusion between division by 0 and ∞, with the two often being used interchangeably.

6(i) This proved to be a straightforward question for many candidates, and the majority gained full credit. Most candidates used the formula for the nth term of an arithmetic progression and another effective method was to generate an nth term expression for the sequence. Informal methods were rarely correct, and the other common error was to use the nth term as $5 + 1.5n$ or even $n + 1.5$.

(ii) The majority of candidates were equally successful here, with solutions being mostly fully correct. Despite being told that it was a geometric progression, many candidates did not recognise w_n as being of the form $a \times r^{n-1}$ and instead generated the first few terms of the sequence to find the values of the first term and the common ratio, not always correctly.

(iii) The majority of candidates could identify that the sum to infinity was required, and correctly state this. There was then some uncertainty as to what was required on the left-hand side, with both the sum of the geometric progression and the nth term of the arithmetic progression being common errors. However many candidates could make a reasonable attempt at both of the summations, but there were a surprising number of errors when attempting to simplify their inequality. The most common errors included only multiplying one side by 2 in an attempt to remove the fraction or incorrect expansion of brackets. Candidates then had to solve the quadratic with both completing the square and use of the quadratic formula being seen, though the latter was by far the most common. A few candidates clearly anticipated that the quadratic would factorise and gave up when they realised that this was not the case. Some candidates, with an incorrect quadratic equation, simply wrote down two solutions with no method shown. In these circumstances, Examiners cannot speculate as to what method may have been used and no credit can be awarded. To gain full credit in this question, candidates had to appreciate that N had to be a positive integer and hence discard their negative root and round up their positive root. Some candidates spoilt an otherwise correct solution by failing to do so.

7(i) The most common approach to this question was algebraic long division, and this was usually successful although some candidates failed to draw attention to the fact that their long division showed them that the remainder was zero. Coefficient matching continues to be a routine method that candidates of all abilities can successfully use, but some omitted to consider a remainder when setting up the identities. A number of candidates started by demonstrating that f(-1) = 0, but never linked this to an explicit statement about the remainder. Some candidates were clearly unfamiliar with the terminology used and made no attempt at the quotient in this part of the question, despite then doing so correctly in part (ii) when it was required as part of the attempt to find the roots. The quotient had to be attempted in part (i) for credit to be awarded here.

(ii) The majority of candidates gained full credit, usually by factorising their quotient from part (i). Some candidates started afresh, either because they did not appreciate the link with the previous part or because an error had been made in the previous part and they realised that the presence of a remainder meant that they did not have a quotient to work with. Some candidates confused 'factors' and 'roots' and their final answer was a fully factorised expression for f(x) with no attempt actually made at the roots.

(iii) Most candidates appreciated the need to differentiate the given equation and could do so correctly. To gain full credit candidates had to equate the derivative to 0 before dividing by 4, and a number of candidates neglected to do this.

(iv) Most candidates could gain the first two marks for correctly integrating the equation of the given curve. The question was testing whether candidates could devise a correct method to find the area of the defined region so no credit was available for integrating f(x) instead. Most candidates could then demonstrate the correct use of limits, although evaluation errors were relatively common, especially when finding F(-1). The method mark could only be awarded if there was an explicit attempt at the correct method, which did not include

candidates whose only method shown was the difference of two incorrect values. Most candidates appreciated the link between this part of the question and the earlier parts and readily attempted to use the correct limits. However, this was not always the case, and some candidates showed admirable, if unnecessary, skill in factorising the given quartic equation. Most of, but not all, the candidates paid due regard to the request for an 'exact' area.

8(i)   Most candidates could correctly identify that it would be a horizontal translation, thus gaining one mark. To gain the second mark, candidates had to state the correct direction and magnitude. The most efficient and convincing method was to use a vector to describe the translation, and a number of candidates did so. However, most candidates opted to use a worded description instead and a number of those failed to gain the final mark as they did not use mathematically precise language. Phrases such as 'in the *x*-axis' were condoned for the first mark, but were not given full credit.

(ii)   Describing the transformation by means of a stretch proved to be much more challenging and only the most able candidates gained any credit at all on this part. Whilst some candidates used index manipulation to rewrite the equation in the form $y = k \times 3^x$ others generated a table of values in an attempt to deduce the effect of the stretch. Otherwise correct solutions were sometimes spoiled by the careless use of language, including both omitting to describe $\frac{1}{9}$ as a scale factor and also an imprecise description of the direction.

(iii)   There were a number of carefully drawn and correct exponential graphs, and many of those with an acceptable graph also correctly gave the *y*-intercept. However too many candidates, who clearly knew the general shape of the curve, were unable to convey their intention in a sufficiently convincing manner. A number of attempts had neither ruled nor labelled axes. A lack of care when sketching the curve resulted in errors such as vertical asymptotes to the right, horizontal asymptotes nowhere near the *x*-axis and curves that had a distinct minimum point in the second quadrant. All of these errors resulted in the mark not being awarded. Several candidates did not extend their sketched curve into the second quadrant, and many candidates did not know the shape at all.

(iv)   Candidates continue to demonstrate proficiency when solving straightforward equations involving logarithms and this was true on this question, with the vast majority of candidates gaining all of the available marks with ease. The most common approach was to use logarithms to base 3, although solutions involving base 10, or even some unspecified base, were also seen. There are still a number of candidates who do not make effective use of brackets, and it was relatively common to see $x – 2 \log 3$ rather than $(x – 2) \log 3$. Some candidates retrieved this by subsequently using their invisible brackets correctly, whereas others continued as if they were never intended.

(v)   This final part of the question was also very well answered, with many fully correct solutions being seen. Candidates generally showed their method clearly, and were able to identify the three relevant *x*-values and attempt the corresponding *y*-values. The trapezium rule was then usually correctly attempted, although some candidates committed the common error of omitting the necessary brackets. Very few candidates attempted to integrate the function before applying the trapezium rule.

9(i)   Only the strongest candidates were able to correctly answer this question. Some had the correct idea, but gave an answer as a range rather than the value of the period as requested. Others could identify the correct transformation but could not then relate this to the period of the function. It was also apparent from comments made that some candidates were not familiar with the term 'period'.

(ii)   This was found to be the most challenging question on the paper and fully correct solutions were in a minority. The most successful approach was the effective use of a sketch graph, as this enabled candidates to identify that the maximum point on this curve occurred at $\frac{3\pi}{10}$, which could then be used to deduce the period of the curve. Others used their graph to establish a relationship between the two roots. An elegant solution, used by some candidates who had clearly studied C3, utilised the double angle formula for $\sin 2A$. Some candidates felt more secure working in degrees rather than radians, and this approach was condoned. It was evident in the solutions seen that most candidates did not consider the possible location and symmetry of the two roots, and simply started with $\sin(\frac{a\pi}{5}) = \sin(\frac{2a\pi}{5})$, resulting in $\frac{a\pi}{5} = \frac{2a\pi}{5}$ from which no further progress could be made.

(iii)   This last part proved to be more accessible to candidates and most were able to make some attempt at the question. Many were able to state a correct equation involving $\tan(ax)$ and then use arctan to obtain a solution for ax. To gain the method mark, candidates had to attempt a solution for x and many struggled to do so. The most common error was multiply, rather than divide, by a and other errors included attempting a solution for a not x and even just stopping at a solution for ax. Candidates who worked in degrees rather than radians could still gain the method mark, and a number benefited from this. Some candidates produced a correct solution to find the first root but then either neglected to attempt the second root or used an incorrect period when doing so. Some candidates attempted to square both sides, but this was rarely successful.

4723 Core Mathematics 3

General Comments

This paper enabled candidates to show their ability in the subject. There were several routine requests which were generally answered well but also some more challenging questions which produced some impressive responses from more able candidates. Indeed it is pleasing to report that over 2% of the candidates recorded full marks on the paper. There were a few candidates who produced relatively little of merit; only 1% of the candidates recorded fewer than 12 marks out of the total of 72. Time did not appear to be a problem; where candidates had not provided any response to a particular request, occasionally the case with parts of question 9, this was thought to be due to the challenging nature of the request.

Two of the questions, numbers 4 and 6, were unstructured questions each worth 8 marks. The approach to be taken in answering such questions is usually evident but many candidates would benefit by taking a moment to plan their way through each solution. This might avoid the situation where solutions sometimes go astray and turn into an aimless set of unconnected calculations.

Many candidates are reluctant to simplify expressions and equations as they go through a solution. The result is that the work then becomes more awkward and the possibility of errors creeping in increases. Examples occurred in question 6. Some approaches reached the equation $36 = 4(4+b)$; many chose at this point not to divide both sides by 4. Another approach in the same question reached the equation $4b^2 + 32b - 260 = 0$; the preferred approach was often to use the quadratic formula with the coefficients 4, 32 and –260.

Comments on Individual Questions

Question 1
Most candidates found this a straightforward opening question and 80% of the candidates duly recorded full marks. The product rule was used efficiently by all but a few candidates. There were some careless errors when substituting –1 in the derivative and a few more as candidates manipulated the equation to the requested form. There were very few instances of candidates using a gradient of $-\frac{1}{12}$ and, in effect, finding the equation of the normal.

Question 2
It was disappointing that this question involving two routine integration requests did not result in greater success. Only 39% of the candidates recorded full marks. The main problem was with part (i) where many candidates did not appreciate that the first step had to be expansion of the integrand; there were many attempts featuring $(2 - \frac{1}{x})^3$. Amongst those who did realise that expansion was needed, there were errors with signs. Candidates fared far better with part (ii) and the only errors to occur with any frequency were an incorrect power of $\frac{2}{3}$ and an incorrect coefficient of $\frac{3}{4}$. One mark was available for inclusion of the constant of integration and most candidates did earn this mark.

Question 3
It was pleasing to see this question on exponential decay handled competently by the majority of candidates; all 6 marks were earned by 78% of the candidates. A minority adopted an approach for part (i) based on powers of 0.8 and this worked well in most cases, just a few multiplying 200 by an incorrect power of 0.8. Most candidates though, perhaps having looked ahead to what was required in part (ii), decided that it was appropriate to establish a formula for *m* in terms of *t*.

They then used this to find the two values in part (i) and to answer part (ii). Usually there was no difficulty in finding the formula although there was some lack of attention given to the signs involved. Some candidates were guilty of having values in the formula that were insufficiently accurate. Lack of care with signs did lead in some instances to a negative value of *t* in part (ii). Other candidates were careless with units, some concluding with 31 seconds in part (ii) and others with 31 grams. These errors with units were not penalised on this occasion.

Question 4
This unstructured question on trigonometry did present more problems to candidates. A few struggled to make any significant progress but the vast majority did realise that they needed to find values of $\tan A$ and $\tan B$. The first equation was the more familiar one and most candidates applied an identity and found the two possible values of $\tan A$ without difficulty. A few candidates went further than necessary and found possible values of the angle *A*.

The second equation was of a less familiar type and many candidates embarked on involved and lengthy attempts. The appearance of $\sec^2 B$ and $\mathrm{cosec}^2 B$ prompted their replacement by $1 + \tan^2 B$ and $1 + \cot^2 B$ respectively. In some cases this led to the correct equation $\tan^5 B + \tan^3 B - 27 \tan^2 B - 27 = 0$ but solution of this equation was beyond most candidates. Those candidates who paused to consider the nature of the second equation in the question observed that replacement of $\sec^2 B$ by $\dfrac{1}{\cos^2 B}$ and of $\mathrm{cosec}^2 B$ by $\dfrac{1}{\sin^2 B}$ offered a more promising approach. Many were able to reach $\tan^3 B = 27$ easily but there were also puzzling cases where an obvious next step was not taken; for example, candidates reaching the equation $\tan^2 B = \dfrac{27}{\tan B}$ sometimes decided to express all in terms of $\sin B$ and $\cos B$. There were errors in reaching the value of $\tan B$ too with values ± 3, $3\sqrt{3}$ and 27 appearing not infrequently.

The identity for $\tan(A - B)$ is given in the *List of Formulae* but care must be taken with signs. Candidates with the correct values for $\tan A$ and $\tan B$ were usually able to conclude the question successfully. There were a few cases where actual angles were used. There were also a few attempts such as $\tan(A - B) = \tan(4 - 3) = \tan 1$ which revealed a basic lack of understanding. Full marks for Question 4 were recorded by 40% of the candidates.

Question 5
In order for particular skills to be assessed, this question required candidates to answer both parts without the use of a calculator. This meant that sufficient detail had to be shown in solutions and many candidates lost some credit for not providing all the necessary steps. The candidate offering only $\int_0^{\ln 2} e^{2x}\, dx = \frac{3}{2}$ as part of the solution in part (ii) did not earn many marks.

Part (i) specified the use of an appropriate equation and therefore candidates showing that both curves had the same *y*-coordinate of 4 when $x = \ln 2$ did not earn the marks. With the answer given in the question, examiners needed to see a convincing solution of the equation $e^{2x} = 8e^{-x}$. For example, solutions going directly from $x = \frac{1}{3}\ln 8$ to $x = \ln 2$ did not earn the second mark. Other attempts showed basic errors in the application of logarithms to both sides of the equation. It was disappointing that only 52% of candidates earned both marks in part (i).

Although 64% of the candidates earned all the marks in part (ii), surprising problems were revealed by some of the solutions from other candidates. The trapezium rule was used in some cases for finding the area under one of the curves and incorrect limits were sometimes seen. There were also attempts to treat the region as one between the curves and the *y*-axis; this is a possible method although it involves integration techniques from Core Mathematics 4 and the attempts seldom succeeded. With the instruction to answer this part without the use of a

calculator, candidates needed to show sufficient detail in their solutions. Most did so although some lost marks through a failure to show clearly how, for example, $\frac{1}{2}e^{2\ln 2}$ becomes 2.

Question 6
Responses to this question varied considerably. Some candidates proceeded logically and efficiently and were able to produce a correct solution in half a page. For other candidates, there seemed to be no plan and solutions meandered on for several pages of complicated algebra. Careful reading of the question was essential and a major problem on many scripts was the failure to form the second equation $18 = a(4+b)^{\frac{1}{2}}$ resulting from the fact that the two curves meet where $x = 2$. Differentiation was often not correct, with the derivative of C_1 appearing as $\dfrac{1}{4x-7}$ a common error.

Candidates who had differentiated correctly and also established the second equation were faced with the equations $18 = a(4+b)^{\frac{1}{2}}$ and $2a(4+b)^{-\frac{1}{2}} = 4$. Some candidates saw that it was then a straightforward process to eliminate one of *a* and *b*; they proceeded to the correct values without fuss. Many other candidates dealt with these equations by squaring both sides of both equations, leading them into quadratic equations with large coefficients and increasing the chances of careless slips. Algebraic errors such as $(4+b)^{\frac{1}{2}} = 2 + b^{\frac{1}{2}}$ occurred far too often for an examination at this level. Some candidates did not help themselves by delaying the substitution of $x = 2$ until a late stage of their solutions.

Full marks were earned by 45% of the candidates but the solutions of other candidates did reveal uncertainties with the necessary algebra and calculus techniques.

Question 7

The response to part (i) was disappointing and 55% of the candidates earned no more than 1 of the 3 marks available. For the first sketch, many candidates seemed content to draw any parabola and there were many attempts consisting of a parabola with its minimum point at the origin. Great accuracy is not required for a sketch but the essential features are expected to be there; in this case the mark was earned for a curve passing through the origin and showing a minimum point in the third quadrant. It was clear that many candidates were unfamiliar with the inverse cosine curve; there were many attempts at the curve $y = \sec x$. For those candidates with the right idea, many did not restrict their attempts to the principal values. Provided the curve was reasonable for the values $0 \le y \le \pi$, the mark was not lost if the curve extended beyond these values. The third mark was available for indicating in some way that the single intersection of the two curves meant exactly one real root; not all candidates with acceptable curves earned this third mark.

There was generally confident work using radians in answering part (ii) and 89% of the candidates earned at least 3 of the 4 marks available. A few candidates made a slip in applying the iterative formula or tried to use a completely different formula. Some did not follow the directions in the question, either giving the values in the sequence correct to only 3 significant figures or, more usually, giving the value of the root as 0.2419.

Part (iii) presented several problems and only 27% of the candidates earned all 4 marks. Many candidates struggled to keep track of various minus signs when giving the equation of the transformed parabola. There was greater success with the other curve and the correct equation $y = -\cos^{-1}(-x)$ was the common response. Some candidates went no further with their attempts at this part and many others evidently expected to be able to solve the equation formed by

equating the equations of their new curves. Candidates who were aware of the geometrical aspects of the question realised that the pair of transformations could be applied to the point of intersection of the two original curves. They had no trouble in identifying the required *x*-coordinate as -0.242 although there were some errors in calculating the corresponding *y*-coordinate.

Question 8

Identifying the range of f was not done well and $f(x) \geq 4a$ was a common wrong response. Candidates generally had more idea with g although some found it difficult to express their answer clearly. Provided the answer conveyed the idea of all real numbers, the mark was earned. The answer $-\infty \leq g(x) \leq \infty$ was frequently offered and accepted. But answers clearly referring to values of *x* were not accepted.

The vast majority of candidates earned 2 marks in part (ii) for finding the inverse of g; the only errors to occur with any frequency were sign slips. The mark for explaining why f has no inverse was not earned so easily. A statement that f is not 1 – 1 or that f is many – one was sufficient to earn the mark. But, in some cases, there was confusion between many – one and one – many. Some responses were contradictory: f is a one – many function, i.e. one value of *y* gives many values of *x*. There were also many comments saying that f has no inverse because of the modulus or that f has no inverse because it cannot be reflected in the line $y = x$.

There was good work seen in response to part (iii) with half of the candidates earning all the marks. The composition of the two functions was almost always carried out the right way round. Dealing with $|2x + a|$ presented some problems. A few candidates immediately replaced it with $(2x + a)^2$; others treated the modulus signs as brackets and proceeded to solve $5(2x + 4a) - 4a = 31a$, an approach which does give one of the solutions of the equation. For those candidates reaching the stage $|2x + a| = 4a$, it was encouraging to note that the majority proceeded without fuss to solve the two linear equations $2x + a = 4a$ and $2x + a = -4a$. Those opting to square both sides of $|2x + a| = 4a$ did not fare so well, not always being able to cope with the quadratic equation involving both *x* and *a*. A few candidates mistakenly decided to reject the answer $-\frac{5}{2}a$ at the end, apparently believing that the presence of modulus signs in the question meant that nothing could be negative.

Question 9

This final question contained some searching requests and it was pleasing to note that 14% of the candidates recorded all of the 12 marks. The majority of the candidates answered part (i) well, providing sufficient detail to convince the examiners.

The three requests in part (ii) made more demands of candidates. The use of 'Hence …' indicated to candidates that the identity proved in part (i) should be used but many candidates appeared to ignore this. Not only was 'Hence …' suggesting the approach to take in each case but it was also indicating that the use of the identity would be the best way to tackle the request. Many candidates made no attempt to use the identity in part (a) and 58% of the candidates scored no marks. Others however used the identity and readily appreciated that the value was

$\dfrac{2}{\sin \frac{1}{6}\pi} + \dfrac{2}{\sin \frac{1}{4}\pi}$, and the required exact value followed.

Some candidates answered part (b) in just a few lines, rewriting the equation as $2\sin 2\theta \cos 2\theta (\tan \theta + \cot \theta) = 1$ and using the identity to reach the equation $4\cos 2\theta = 1$ followed by the value of θ. Many candidates though clearly believed that the equation could not be solved unless the equation was expressed in terms of $\sin \theta$ or $\cos \theta$; involved attempts followed using various identities and sometimes the attempt was concluded correctly.

Only 21% of the candidates answered part (c) correctly but it was pleasing to note neat and elegant solutions such as

$(1-\cos 2\theta)^2 (\tan\frac{1}{2}\theta + \cot\frac{1}{2}\theta)^3 = 4\sin^4\theta(\tan\frac{1}{2}\theta + \cot\frac{1}{2}\theta)^3 = 4\sin\theta[\sin\theta(\tan\frac{1}{2}\theta + \cot\frac{1}{2}\theta)]^3$ and the use

of the identity reduces this to $4\sin\theta \times 2^3$ and therefore $32\sin\theta$.

4724 Core Mathematics 4

General Comments:

The paper proved accessible to almost all of the candidates, but there was also enough to stretch the best and full marks was rarely achieved. There were many examples of well-presented responses and some excellent work was seen by most examiners. However, there were also many instances of poorly presented work and handwriting that was very difficult to read – in some of the more extreme cases it seemed that candidates couldn't read their own writing as there were many examples of miscopied work.

Many candidates demonstrated a good understanding of Core 4 material, but failed to do themselves full justice in the examination either because they didn't answer the question fully or because of poor algebra – bracket and sign errors were common - and careless arithmetical slips.

A surprising number of candidates were unable to relate work done in the early part of a question to the demand of a subsequent part. It is advisable to read through the whole question first rather than attempt a solution in a piecemeal fashion.

Comments on Individual Questions:

Question No. 1
This proved accessible to nearly all candidates, with most scoring full marks. Most opted for long division and were successful, although a few made sign errors. Various other approaches were also seen: by and large they were successful too.

Question No. 2
Most candidates realised the need to use the appropriate double angle formula and successfully integrated to obtain an expression involving $\sin 8x$. Sign errors were quite common, however, and $12x$ was commonly seen. A few candidates worked with $\cos 2x$ and didn't score. A significant minority had no idea how to deal with $\cos^2 4x$ and tried to integrate directly.

Question No. 3
This question was done very well indeed, with many candidates achieving full marks. A common error was to differentiate the second term as $\ln x$ and some candidates made sign or coefficient errors when using the product rule.

Question No. 4
Most candidates knew how to integrate by parts, but many made accuracy errors, particularly when dealing with the second integral. Some candidates worked carefully through the problem, but either didn't see the instruction to leave the answer in terms of $\ln 2$, or didn't know how to resolve $\ln 8$.

Question No. 5
Part (i) was done very well indeed. Most candidates found s and t successfully and went on to find the point of intersection and confirm that all three equations were consistent. A few candidates lost an easy mark by omitting one of the last two requirements.
In part (ii) many candidates successfully demonstrated the equivalence of the direction vectors, but rather fewer appreciated the need to identify a common point.

Various other approaches were seen but in many cases candidates left rather too much to the imagination of the examiner to earn full marks.

Question No. 6
Most candidates understood the drill for integration by substitution, and most laboriously expressed dx in terms of u and du rather than factorising the numerator and cancelling out $2x$. The method marks were achieved by most, but poor algebra often led to the loss of the accuracy marks. Of those who successfully integrated the correct expression in u, a significant minority lost the final accuracy mark by omitting "+ c" or by failing to substitute back in terms of x.

Question No. 7
Many candidates knew what to do here. Most wrote down the correct equations and successfully substituted to eliminate either k or n. Many candidates made basic errors in the manipulation of the equations such as the substitution of $n = -k - 6$. Even when substitutions were correct, the algebra often went wrong. Many candidates either neglected the request to state the set of values for which the expansion is valid, or demonstrated a variety of misconceptions. Answers ranging from "x is any real number" to mod $x < -$ ¼ were seen.

Question 8
Part (i)
Nearly all candidates understood how to use the scalar product to solve this problem, and many went on to score full marks. A few lost an accuracy mark because they were unable to find the larger angle correctly, and some failed to spot that a double angle formula was required.
Part (ii)
This proved more demanding. Many candidates found the correct lengths and then automatically used ½absin$C$. Some lost accuracy through working with rounded numbers, or through bracket or sign errors in substitution, but a significant minority went on to score full marks. Some candidates showed very little working and achieved a wrong answer, thus denying the possibility of earning any method marks.

Question 9
Part (i)
Most candidates set $y = 0$, but few went on to successfully find all three values. Surprisingly, (0, 0) was almost as commonly omitted as (1, 0) and (2, 0).
Part (ii)
Many candidates knew what to do to obtain the required result, and there were many examples of clear, well-structured solutions. Most realised the need to resolve the double angle for the next part of the question, and many made no further progress. Only the best candidates went on to obtain both pairs of coordinates correctly.
Part (iii)
A significant number of candidates worked immediately with inverse trig functions and failed to score – not realising that they could not achieve a polynomial expression by this route. Many candidates appreciated the need to use the double angle formula and Pythagoras, but mistakes in expanding brackets were common and the "2" was frequently omitted following substitution. Expressing cos$t$ in terms of x often went wrong and the high frequency of algebraic slips prevented many candidates from achieving full marks.
Part (iv)
Not many candidates made the connection between this part of the question and earlier work. Cubics of the right orientation and with the right intercepts were sometimes seen, but very few candidates appreciated the restriction on the x-values. Nevertheless, a few candidates who had made little progress in earlier parts of the question reached for their graphical calculators and achieved both marks.

Question 10
Part (i)
Most recognised the correct form of partial fractions and successfully cleared the fractions to produce a fully correct solution.
Part (ii)

The separation of variables caused problems for many, but most recognised the link with part (i) and worked with their partial fractions. The method was often well understood, but only a minority of candidates had the stamina and attention to detail to go on to the end and achieve the correct result.

4725 Further Pure Mathematics 1

General Comments:

The majority of candidates were able to make a reasonably good attempt at a good number of questions, showing a good range of knowledge of many parts of the specification. Completely correct solutions to all questions were seen, and no question proved inaccessible to the candidates. The standard of presentation appeared to be better this year, but poor handwriting often lead to marks being lost through simple arithmetic or algebraic errors when the correct method was known.

There was no evidence of candidates being under time pressure, with most making some attempt at the majority of the questions. However, as has been mentioned in previous reports, in a number of questions there was the opportunity to check a solution, but few candidates did this and so could not identify that an error had occurred and have the chance to rectify it.

Comments on Individual Questions:

1 Most candidates expanded and attempted to use the correct standard results, although some used only the formula for $\sum r$ and then expanded. Many made algebraic slips, especially in the middle term, which lead to an expression that could not be factorised and many made an error when trying to take out a factor of ½.

2(i) Those candidates who sketched an Argand diagram and used trigonometry usually found the correct value for z. Candidates who gave equations for the modulus and argument often made a sign error, not seeing that z is in the 4th quadrant.

2(ii) The conjugate and the method of rationalising was shown by most candidates, but again errors in arithmetic were frequent, with e.g. the value of the denominator, 49, being found as 47.

3(i) This was generally answered correctly, sign slips or failing to include the coefficient of x^2 being the most common errors.

3(ii) Expansion of the given expression and using the results from (i) was the most popular method, with most being able to deal with $\alpha^2 + \beta^2$ in a sensible way. A significant minority saw that $\frac{1}{\alpha} = \beta$ and $\frac{1}{\beta} = \alpha$ and so could find the required value in a very neat way.

4(i) Most candidates found the correct answer, with $5a - 3b$ becoming $2a$ the most common error.

4(ii) Many candidates thought this product gave a 3 × 1 or a 3 × 3 matrix. Those who correctly recognised the answer was a 1 × 1 matrix often omitted the matrix brackets.

4(iii) The correct 3 × 3 matrix was usually found.

5 Some candidates failed to show clearly that the result is true for $n = 1$, while others started the process at $n = 2$. Most were able to show that using the given form in the recurrence relation set up the induction process correctly. A significant number of candidates did not give a clear explanation of the induction process, so the last mark was often lost.

6(i) There was a certain amount of confusion between the Cartesian and complex forms here. The centre was often written as (3, 3i), while the complex form of the circle was given as $(x-3)^2 + (y-3i)^2 = r^2$.

6(ii) The sketching was not particularly good, omission of any scale or coordinates was quite frequent, while the circle often crossed either or both of the axes. The line *l* was often drawn at the ends of the diameter, rather than through the centre of the circle.

6(iii) Many candidates found the Cartesian equations of *C* and *l* correctly and then solved to find the correct points of intersection as coordinates, but then failed to give these points as complex numbers. Those who had drawn a good sketch were often able to find the points of intersection correctly with little effort.

7(i) Most recognised that the transformation was a shear, but often did not clearly state that the *x*-axis is invariant or give a suitable image point under this transformation.

7(ii) The most common error was using **M** = **QP**. The resulting matrix for P is then clearly not a single transformation, but candidates did not see that an error had probably occurred.

8(i) Most showed sufficient working to justify the given result. A numerator of $2r + 3 - 2r + 1$ was the most common error.

8(ii) The method of differences was understood by most candidates. Many did not give the answer as a single fraction and a good proportion failed to include the required factor of ½ in their final answer.

8(iii) This part proved quite testing. Most candidates tried to find the sum to infinity but many got 0 or 1 for this and many subtracted the sum to *n* rather than the sum to *n* – 1. Again the factor of ½ was often omitted.

9(i) The method for finding the determinant of a 3 × 3 matrix was demonstrated by virtually all candidates.

9(ii) Many candidates simply stated that det **X** = 0, without showing that this is true for the value *a* = 3. Most candidates attempted to solve the system of equations, but many made algebraic slips and so could not deduce the correct conclusion.

10(i) Most candidates were able to show the correct method for finding the square roots of the given complex number. Sign errors caused the greatest loss of marks, and very few checked their answers by squaring the results that had been found, a simple process that would have highlighted an error in the working.

10(ii) Insufficient working often caused the loss of this mark. Those who substituted into the given quadratic omitted to show the expanded version, while those who used the sum and product of roots approach often only used one of the two results. Errors in using the quadratic formula were also quite common.

10(iii) This part proved quite taxing. Many did not show a convincing connection between the *z* and *u* equations. Only a small proportion of candidates realised that the quartic equation had four roots, the other pair being easily derived from the conjugate root, 9 – 40i.

4726 Further Pure Mathematics 2

General

There were a similar number of candidates this year to last year and the mean mark was marginally lower. Most candidates had clearly covered the syllabus, though with varying degrees of understanding.

There was a demand in questions 3, 4 and 6 for a sketch of a curve. We felt that these sketches left a lot to be desired and many lost marks because the essential feature for which marks were being awarded could not be seen. Comments have been made in the specific comments section below.

In question 7 there was a hint that candidates would find it helpful to sketch the curve and rectangles; many did but with little understanding so once again essential features could not be seen.

Question 1
This question provided an easy source of marks for all but the weakest of candidates at the start of the paper.

(i) Candidates were advised to begin by expanding $(e^x + e^{-x})^3$ and the majority did as they were instructed. The vast majority achieved, eventually, the required expansion and it remained to convert this expression into the required identity. There are, of course, many different approaches to achieve this but there are one or two general rules to be borne in mind:
- state the result you are trying to prove either at the beginning or the end.
- make clear what the terms being used represent rather than simply manipulate a range of expressions
- avoid a style of proof which end with 0 = 0 QED

There were many correct solutions seen although a proportion of these could scarcely be described as succinct.
There were other approaches that could lead to success although those employing cosh(A + B) etc would be well advised to **know** the correct expression.

(ii) Most candidates used the result proved in part (i) to form a cubic equation in cosh$x$; factorisation was duly carried out and the three values found. Two of the values for cosh$x$ were 0 and −1.5 and it was important that candidates indicated that, since cosh$x \geq 1$, these answers were not valid. A number of candidates either ignored these values without explaining why or tried to find x-values corresponding to either 0 or −1.5.
Those who arrived at the only solution of cosh$x$ = 1.5 were often able to convert this to
$\ln(\frac{3}{2} \pm \frac{\sqrt{5}}{2})$ although the second root was frequently omitted.
It was also possible to express the equation in exponentials but this route proved much more complex and those who tried this approach were often unable to form and solve the resulting cubic in e^{2x}. Many of the weakest candidates expressed the equation in exponentials but were unable to make further progress.

Question 2
The partial fractions part was nearly always attempted in the correct manner with almost all of these then going on to the correct result. Many candidates did not explicitly state the partial fractions but this was condoned if they were then seen in the integration. A surprisingly high number of candidates did not spot the arc tan integration and gave some version of ln instead.

Given that this is a FP2 paper they should expect to be integrating functions that are uniquely in this specification.

Question 3
Candidates found the graph hard to draw accurately but most were still able to achieve the majority of the marks.
Almost all recognised that the graph needed to be symmetrical about the *x*-axis even if none of the other criteria were met. Similarly, almost all recognised the ranges in which the graph existed and many of those whose first step was to shade out the invalid ranges went on to achieve full marks. It seemed that many candidates were unaware of the need for the graph to be parallel to the *y*-axis as it crossed the *x*-axis or that it would meet the original *y* = f(*x*) curve when *y* = 1. There was a link between these two criteria, for if either one of these two features had been appreciated, then the other mark would almost certainly have been gained as well. Many candidates with otherwise correct graphs had not realised that the question had asked for the co-ordinates of the points where it crossed the axes to be stated explicitly. For those that had done so there were still some common mistakes including

- Omission of $(0, -\sqrt{2})$ entirely
- $(0, 1)$ and $(0, -1)$ instead of $(0, \sqrt{2})$ and $(0, -\sqrt{2})$
- $(\sqrt{5}, 0)$ instead of $(5, 0)$
- Even on a correct graph $(0, 2^2)$ and $(0, -2^2)$ instead of $(0, \sqrt{2})$ and $(0, -\sqrt{2})$ which was a very surprising error but seen on a number of occasions.

Those that drew the two sections reasonably well may have appreciated that the curves crossed at three places on the line *y* = 1 but it was not obvious on their sketch – a case where a more carefully drawn sketch could have earned more marks.

Question 4
There were many good answers to this question; candidates usually find the questions on this topic straightforward.

(i) It was rare to find a script in which this part did not yield the one mark.

(ii)(a) As with the other questions, the drawing of the graph was not good. Many failed to appreciate the fact that $x = \frac{1}{2}$ was an asymptote and so struggled to draw what they thought was the correct graph that fulfilled the result of (i). A significant number of candidates drew the section where the curve approached the asymptote as a "spike" (from graphical calculators without thinking about the nature of the curve?). Candidates who rewrote the expression as *y* = 2ln(2*x* − 1) lost the part of the graph for $x < \frac{1}{2}$. The best candidates marked the asymptote using a dashed line.

(ii)(b) The 'staircase' needs to move vertically to the curve and horizontally to the line and there needed to be some indication of the start (labelling x_1 or a vertical line starting on the *x*-axis or arrows).

(ii)(c) The fixed point iteration was very slow and it needed at least 11 iterations to achieve accuracy to 3 decimal places. Most candidates did quote their answers to 3 decimal places but did not achieve the accuracy in the 3rd decimal place that was required.

(iii) Newton-Raphson has a much faster rate of convergence and β to 5 significant figures was achieved very quickly. Candidates should check the accuracy required in the question; some gave their answer to 5 decimal places instead of 5 significant figures. It should be noted that the correct Newton-Raphson formula with the correct f'(*x*) was required. Some candidates rewrote the equation in a different way, achieving an *x* = g(*x*) iterative formula that converged to β.

However, this is not the Newton-Raphson formula that was required here and so merely finding β did not earn any credit. Likewise an incorrect f'(*x*) will result in an iterative formula that converged to β, but again this was not the Newton-Raphson method and so earned no credit. Both of these erroneous processes to find β demonstrated a very much slower rate of convergence – the initial value *x* = 1.6 is already accurate to 1 decimal place and so accuracy to at least 4 decimal places is achieved in only 3 iterations using the Newton-Raphson formula.

Question 5
(i) There were a number of methods employed to differentiate $\tan^{-1} 2x$:
* use the standard result from the formula sheet plus chain rule,
* rearrange to tan*y* = 2*x* and differentiate implicitly,
* rearrange to $x = \dfrac{1}{2\tan y}$ and find $\dfrac{1}{\dfrac{dx}{dy}}$.

Almost all candidates obtained the required answer although the numerator of 2 was missing from those who found problems using the chain rule.
The second differential posed few problems for most and then followed the algebraic proof to the required answer. Candidates should be aware that proofs of this type must contain no algebraic errors and it was a little surprising that a significant minority made careless slips; in almost all of these cases the algebraic slip was then mirrored in the compared expression.

(ii) The expression required in part (i) was intended as a hint for candidates as to the most efficient form for finding the third differential; most ignored the suggestion and used the quotient rule to attempt the differentiation. Unsurprisingly, there were a considerable number of slips when handling this messy expression and it must be appreciated that errors at this stage will be penalised in the later part of this solution. In the final Maclaurin expansion it is expected that all

coefficients are presented in their simplest form and answers such as $\dfrac{-16}{3!} x^3$ are not

acceptable.

(iii) This short question on using the series appropriately was not well done. Most realised that

$\tan^{-1}\left(2 \times \dfrac{1}{2}\right) = \dfrac{\pi}{4}$ was required but ignored the word *only* which implied that not only should

the values be identical when rounded to one significant figure but that they be different when rounded to two significant figures. Only the very best candidates were able to appreciate that this extra element was required.

Question 6
(i) The majority of candidates scored full marks. The most common mistake was to place the loop too high in the quadrant. Stronger candidates were clearly making an effort to make $\theta = 0$ a proper looking tangent whereas others seemed to have a shaky concept of what that meant.

(ii) An equation was asked for and most candidates followed this instruction as they did for giving coordinates with most favouring a decimal format.

(iii) Most candidates recalled the formula correctly and were able to use their trigonometrical identity knowledge to convert the function into an integrateable form. The most common mistake was to get the sign wrong either in the identity or in the integration.
A significant minority attempted integration by substitution or by parts and did not progress further. Candidates should be advised to make sure they are familiar with the trigonometrical identities from the core modules.

Question 7
(i) Those candidates who produced concise, efficient solutions almost invariably started with a well drawn diagram where the heights of the rectangles were clearly equivalent to equally spaced ordinates. Many candidates produced unsatisfactory diagrams with seemingly random rectangles placed above the curve and, although they recognised that their summation was greater than an integral they made heavy weather of producing a fully correct solution. Whilst many candidates who did not draw a diagram were also able to get a full solution, these tended to start off by looking at the area of n rectangles compared to an integral between 1 and $n + 1$ (or even just n) before moving on to look at the infinite case thereby making their solution lengthier. Some candidates started off by comparing a specific number of rectangles with a corresponding integration and continued to work numerically. Few of these went on to successfully extend to the infinite case.

Many other candidates drew rectangles below the curve and argued that since their integration gave an infinite value then an infinite series of rectangles that were being used as an approximation must also be infinite, some of these actually went on to do part(ii) correctly and yet did not realise that this highlighted the fallacy in their reasoning.
Other candidates drew two series of rectangles, one above and one below the curve, clearly believing that it was necessary to provide upper and lower bounds.
Nearly all candidates were able to carry out the integration correctly.

(ii) As in part (i) those candidates who started with a well drawn diagram tended to get a complete solution with very little extra work required. Adding in the extra 'first' rectangle and shading it was a commonly used effective strategy and for most successful candidates the '+1' required appeared early in their working rather than towards the end. There were some candidates, however, who would add 1 to the result of their integration but without any apparent understanding of why. It is possible that they had realised that $1+\dfrac{1}{4}+\dfrac{1}{9}+\dfrac{1}{16}+........$ was clearly > 1 and probably < 2 so it was a good thing to do or it may have been that they had a recollection of using this type of method in other examples.
Many candidates who had drawn rectangles under the curve in part(i) reversed their error in part(ii) and drew them above this time.
As in part(i) two series of rectangles were often seen.

A worryingly large number of candidates integrated incorrectly to get $\left[-2x^{-3} \right]_{1}^{\infty}$; by coincidence this gives them the answer of 2 immediately which may have convinced them they were correct at that point. Others carried out their integration with 2 as the lower limit (easily confused with the 2 for their first rectangle) and obtained an answer of ½ which then becomes 3/2 once they add on their extra rectangle (which they all did since they weren't going to use 2 without appreciating the need for an extra rectangle).

Question 8
Some candidates seemed to run out of time (or inspiration) but most made an attempt at this question.

(i) Most candidates realised that they needed to use integration by parts but some used $\sec^2x = 1 + \tan^2x$ before integrating, which made the problem much more difficult and with the wrong choice for "u" and "v" impossible. A small number of candidates split the parts correctly but were not able to differentiate $\sec^{n-2}x$ correctly, often saying that $u' = (n - 2)\sec^{n-3}x \tan x$ instead of $(n - 2)\sec^{n-3}x \sec x \tan x$, leading to an expression for I_n that involved I_{n-1} and I_{n-3} instead of I_n and I_{n-2}.

(ii) Whether the recurrence formula had been achieved or not, most candidates used the given result to connect I_8 to I_6, I_4 and I_2. The majority of the candidates were able to show that $I_2 = 1$ and hence deduce $I_4 =\dfrac{4}{3}$, $I_6 = \dfrac{28}{15}$ and $I_8 = \dfrac{96}{35}$.

(iii) The initial case for the induction followed (since I_2, or I_4, is rational). The most successful candidates were those who set out their induction formally, stating that the assumption was that I_{2k} is rational and then using the recurrence formula to relate I_{2k+2} to I_{2k}, together with stating that $\sqrt{2}^{2k} = 2^k$, to reach the required conclusion. Far too many candidates did not discuss the nature of the $\sqrt{2}^{-2k}$ term. It was disappointing to see so many candidates write "assume true for $n = 2k$" but fail to state that they now needed to show that it was also true for $n = 2k + 2$ and wrote down the reduction formula from (i) for $n = 2k$ and this caused them confusion regarding what they needed to do.

4727 Further Pure Mathematics 3

General Comments:

Overall this paper was found to be slightly harder for well-prepared candidates than recent ones, although it still produced a good spread of marks. Most candidates were able to attempt all questions. Many of the questions allowed weaker candidates to demonstrate basic techniques, but also contained parts to stretch the most able, particularly in terms of problem solving

On questions where demonstration or proof was required many candidates are not providing enough detail in their solutions. Question 7 (ii), with the many possible approaches, would make an excellent starting point for Centres wishing to develop a discussion of rigour.

Group theory remains the topic for which candidates would benefit from better preparation from Centres, but there was more this year on which marks could be scored more easily, especially on the tables in 8 (iv).

Many candidates lack the ability to use precise mathematical language when they are producing mathematical arguments, and this is another area where Centres may wish to devote more time to developing this skill.

There were many candidates who had been well prepared for this exam with a sound knowledge of each topic area; however there were once again some candidates who were unable to tackle questions on whole areas of the syllabus.

Comments on Individual Questions:

Qu. 1

(i) Most candidates were clearly well versed in finding roots of unity. It is expected that the real root, 1, should not be left in exponential form. Those losing marks had sometimes misunderstood the instruction to give all **non-real** numbers in the given form resulting in them losing a mark due to only giving 4 roots. A few others completely omitted *i* in the exponents consequently scoring zero.

(ii) The best answers gave the roots in precisely the required form and then produced 5 equally spaced, labelled points or vectors on an Argand diagram with labelled axes (Re, Im). The majority of candidates gained at least M1 for -2 times their roots of unity, and many successfully converted to the polar form with $r = 2$. Some, erroneously, simply tried 2 times the roots of unity or, realising that they had to deal somehow with the -32, used $2i$ times the roots of unity. Most candidates gained 1 of the 2 marks available for their Argand diagrams due to at least having located their roots in each quadrant; however, many were scrappy, with badly unequal lengths, angles or with asymmetric complex conjugate roots.

Qu. 2
The majority of candidates had learnt the standard method given in the mark scheme for finding the shortest distance between lines and, apart from minor slips, they were able to handle this question with ease. Another fairly common, successful approach was to convert the problem into one of finding the shortest distance between two parallel planes each containing one of the lines. A valid alternate approach which was sometimes seen involved using 3 parameters – one for each line plus one for the vector joining them perpendicularly. Most candidates attempting this approach were unable to complete it accurately. Some candidates attempted to use the standard method, but without secure knowledge of which vectors went where within the formula.

Qu. 3

(i) Most candidates were able to use implicit differentiation to find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of u, and then substitute efficiently and clearly into the original equation. Some, instead, found $\dfrac{\mathrm{d}u}{\mathrm{d}x} = -\dfrac{1}{y^2}\dfrac{\mathrm{d}y}{\mathrm{d}x}$ $\frac{du}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$ and correctly substituted for this whole right-hand expression. Candidates should be discouraged from writing meaningless expressions like $\mathrm{d}y\!=\!-u^{-2}\mathrm{d}u$ $dy = -u^{-2}du$ when constructing a demonstration, although their (otherwise valid) argument was not penalised in this instance.

(ii) Most candidates used an integrating factor to effectively solve this first order differential equation but a significant number failed to evaluate the constant of integration. A few were able to convert the equation into integrable form by inspection alone. Of those who were unsuccessful, a common error was to multiply the RHS of the **original** equation by x^2, resulting in the incorrect equation $\dfrac{\mathrm{d}}{\mathrm{d}x}(x^2 u) = 1$. Another not infrequent fault was to arrive correctly at $\dfrac{1}{y} = \dfrac{\ln x}{x^2} + \dfrac{A}{x^2}$ and then invert it to $y = \dfrac{x^2}{\ln x} + \dfrac{x^2}{A}$.

Qu. 4

(i) Clear arguments were key to gaining the marks in this part. The best answers first explicitly identified the identity as 1, and then either used a single counter-example to show that the set did not contain all inverses, or generalised by considering the inverse of a, for $a \neq 1,-1$. . Those who just said that $\dfrac{1}{a}$ is not an integer did not gain the second mark, unless they demonstrated a fuller understanding by at least excluding $a = 1$.

One misconception, which often escaped penalty due to sufficient correct reasoning, was the belief by some candidates that a lack of inverses within the set also demonstrated set closure.

(ii) Many candidates answered this very well, correctly identifying $\{1,-1\}$ and itemising the four requirements for a group, closure usually being shown by a table, and the inverse of -1 being stated as -1. Historically, candidates sitting this paper have not always appeared aware of standard set notation, but this year it was rare to see sets described without the use of the correct style of parentheses. There was, however, still some confusion between associativity and commutativity. The most common error was to state that the group was $\{1\}$

Qu. 5

Most candidates had been well prepared for solving differential equations. The most common errors occurred in calculations made whilst solving the simultaneous equations. The ability to use a calculator which solves simple equations would obviously have been helpful to those prone to this type of error. Other occasional errors seen in scripts included using a complementary function involving a term in $i \sin 3x$ or $e^{-1}(A\cos x + B\sin x)$ and using the trial function $y = a\cos x$. There were still some occasions where candidates omitted "$y =$" from their general solution. Some candidates used θ instead of x and hence lost marks.
A few left the CF in complex exponential form: the Examiner was keen that candidates evidence their understanding that a real solution exists.

Qu. 6

(i) A lot of candidates gave the vector equations of the line instead of its Cartesian form. Most candidates successfully found the direction of the line, using the relevant vector product, and then found a particular point on the line. Those who tackled the problem by solving Cartesian equations often made numerical errors along the way. Generally, in this paper, fractions should not occur within the numerator or denominator of a fractional answer and the denominator should not be negative. Here, due to the significance of each part of the equation, the answer was expected in a form such as $\frac{x-\frac{7}{3}}{7} = \frac{y-\frac{1}{3}}{-2} = \frac{z}{-3}$. Other correct answers of the form f(x) = g(x) = h(x) were also accepted.

(ii) Often candidates first showed the relevant scalar product to be zero and concluded that the line and plane were parallel. (They were expected to show the workings from their scalar product). Some then tested one point of the line, such as $(0,1,1)$, in the plane equation, and concluded that the line did not lie in the plane. But very many here went back and substituted the general parametric point into the plane equation, reaching the same conclusion. Such candidates failed to appreciate that this rendered the earlier part of their answer redundant.
A few candidates merely substituted parametric equations into the plane to get an inconsistent equation and claimed, correctly, that this was sufficient demonstration.

(iii) Verifying that the point lies on each plane did require at least some workings – even though these workings are extremely simple ones. Mostly this was done efficiently.
Since the line of intersection of these two planes is parallel to the line in part (i), there is no need to do another vector product (the planes geometrically formed a prism). While many candidates got full marks, few used the above fact and wrote the answer down without further calculation. The vector equation should begin with "$\mathbf{r} =$", whereas a significant number of candidates either omitted this or wrote "line =" or "$l =$".

Qu. 7

(i) Where candidates produced good solutions, these were well laid out, with precise use of the equals sign; their scripts clearly referenced the application of De Moivre, showed their full binomial expansion, were explicit about taking the imaginary part, and showed how they replaced terms in cosine with equivalent ones in sine. While the use of *c* and *s* is acceptable in intermediate calculations, the letters should be defined at the start and not retained in the final answer. Many candidates failed to include this level of detail, whilst still accessing some of the marks available. Centres should note that conventionally (as given in Appendix B – Mathematical Notation) Im(z) = y and not iy. Where candidates misapplied this convention, they were given the benefit of the doubt on this occasion.

(ii) Most good solutions to this question resulted from completing the square or from simplifying the function into one involving multiple angles. Once either of these steps was taken, candidates were much more capable of convincing with their justifications. Many made some progress by differentiating with respect to θ and equating to zero, but then usually failed to find all stationary points or to convince on their nature. Still others made the substitution $x = \sin\theta$ and then treated the expression as a quadratic, but then failed to take account of the extra complexities due to the range of x being restricted to $[1, -1]$. Another approach which candidates used when trying to find the minimum was to consider the intersection of the function with the line $y = k$, and finding where the discriminant of the resulting equation was non-negative. Many candidates merely worked back from the given answer, and scored only the first mark (for using $\sin 2\theta = 2\sin\theta\cos\theta$ to simplify the initial expression). Some candidates equated the function to zero for some reason and then factorised this expression, to no real benefit.

Qu.8

Good scripts always made clear that their starting point was an assumption (Let ..., Assume..., or similar) and also concluded their mathematical arguments with a statement of what had been shown. Both steps are important when conducting proof by contradiction. Whilst omission of the former step was condoned on this occasion, the latter was strictly insisted upon by the Examiner. Some candidates had clearly never met proof by contradiction, often starting with a statement of non-equality.

(i) Strong answers either used proof by contradiction, or considered each value of n and justified

their rejection. These answers were marked by conciseness and a statement/s concerning the

distinctness of the set elements defined by the question. Some otherwise good answers lost the

final mark by omitting to conclude their argument with "So $ba \neq a^n$". Many candidates missed

the point and talked only about what happens when you multiply *a* by itself repeatedly.

(ii) This question proved beyond the capabilities of most candidates, and although all the different marks available were accessed by some candidates, it was rare to see them all combined to form a complete proof. Good answers to the proof that $ba \neq a^2b$ came from candidates who persevered to find a contradiction.
Most candidates tried to prove that ba was not equal to ab by element operation, or presumed that the non-commutative nature of the group meant that no two elements in the set commuted with each other, rather than using the fact that if *ab* were equal to *ba* then all element pairs would consequently have to be commutative.
Many candidates sensibly attempted to deduce that $ba = a^3b$ by eliminating other options, but most failed to notice that they had to rule out *b* as a possible candidate for *ba*.

(iii) Good answers to this question started with one side of the identity to be proved and used a clear chain of steps linked by equals signs to the other side of the identity. Many candidates demonstrated poor mathematical language by starting with $ba^2 = a^2b$ and continuing through steps to show that this implied that $a^2b = a^2b$, or even worse that e = e.
 Some candidates unsuccessfully attempted to show this identity without using the fact that $ba = a^3b$.

(iv) There was evidence that most candidates had some familiarity with the structure of groups of order 4 in that most were able to gain some marks here. It is clear that some candidates have been well drilled in the cyclic and Klein patterns and were able to gain most marks here even when they had been unsuccessful with earlier parts of the question. The cyclic table was most often found, although even this occasionally had incorrect elements within it. Many also found at least one of the two Klein groups $\{e, a^2, b, a^2b\}$ and $\{e, a^2, ab, a^3b\}$ although it was not uncommon for the leading diagonal to mistakenly contain elements other than the identity element.

4728 Mechanics 1

General Comments:

Candidates were well prepared for the routine parts of the paper. Questions demanded use of the full range of constant acceleration formulae, and candidates coped very well with these. The final question of the paper, dealing with variable acceleration kinematics, demonstrated that the majority of candidates could cope well with this area of the syllabus, though there was a suggestion that integration was less thoroughly known than differentiation.

The greatest difficulty faced by candidates was the understanding and interpretation of individual tasks. This was most clearly seen in Q3, Q4(iii) and Q6. Fuller details of the problems encountered here are given below.

Many instances were seen of candidates failing to evaluate expressions accurately. There was also a tendency to rearrange formulae before substituting numerical values; this risks a loss of marks as method marks usually reflect correct substitution of numbers into valid formulae. There was no evidence of candidates having insufficient time to complete the examination. Some re-reading of questions (to ensure they had been understood correctly and answered in their entirety) would have benefitted candidates.

Comments on Individual Questions:

Question No. 1

(i) Most candidates scored full marks for this part, and only a few gave only one of distance and time.

(ii) Again most candidates scored full marks.

Question No. 2

(i) Most candidates scored full marks. Usually appropriate *suvat* equations were selected, though a minority of candidates solved (correctly) two simultaneous equations.

(ii) By far the most common approach error was to give the angle with the horizontal, so 2/3 was the most frequent score. A significant minority of candidates did not link the acceleration in (i) to the inclination in (ii).

Question No. 3

(i) The simplest solution (resolving parallel to the bisector of the required angle) was seldom seen. Resolving parallel and perpendicular to one of the 4 N forces was more common. Both approaches were generally successful. However, using the cosine rule on an incorrect diagram which reflected subtraction – not addition – was by far the most frequent approach, and led to candidates usually obtaining 2 marks out of 4.

(ii) Candidates needed to appreciate that the 6 N resultant would be perpendicular to the smooth surface on which the particle rested. This would be demonstrated by having the angle in (ii) equal to the complement of half the angle in (i), for which a mark was awarded. The value of *m* could be determined using the given 6 N resultant. So candidates who resolved 4 N forces (using an erroneous answer for the angle in (i)) gained a method mark, but not an accuracy mark. Some candidates lost a mark through giving the mass as 0.92 kg, rather than 0.918 kg.

Question No. 4

(i) This was almost always answered correctly.

(ii) This was correctly answered even more often than part (i).

(iii) This was rarely answered well, using the understanding that the speed of *B* after the third collision must be at least $4\,\text{m}\,\text{s}^{-1}$, if it is not to be struck again by *A*. The most common assumption was that *B* would come to rest if the combined particle *C/D* were to have its greatest speed. The second most common assumption was that *B* and *C/D* would coalesce.

Question No. 5

(i) A good diagram showing all forces was beneficial to those who included it. The only common error was that the squaring of the $2P$ N force when using Pythagoras theorem was written $2P^2$. Too often candidates failed to see their "invisible brackets", and obtained a quadratic equation starting $3P^2$ which would fail to give a useful answer. Several candidates found the angle the resultant makes with the horizontal at the end of part (i).

(ii) Many candidates made a reasonable attempt, but fully developed solutions were rare. Using the vertical component of the resultant to find the normal component of reaction (often ignored), and then using Newton's Second Law with the correct forces was demanding. The direction of motion was presented in many ways, but only "East" or "Bearing 090°" (using the context set in the question) were deemed acceptable.

Question No. 6

(i) Though many good solutions were seen, many candidates sought to solve this part of the question without reference to acceleration. The tension in the string before release was often correct, though candidates who were mislead by the reference to particle *P* being held lost the mark available. Setting up and solving two simultaneous equations in *T* and *a* was necessary for the "after" motion, but the solution leading to $T = 1.66$ N (based on $a = 0$) was often seen. A second common error was to simplify $T – 0.98 – 0.679$ (forces on *P* parallel to the slope) to $T – 0.301$.

(ii) It required a major shift in thinking for candidates to realise that values for *T* and *a* worked out in part (i) would be no use in part (ii). Many did so successfully, including those who had erred in (i).

(iii) Though "Contact Force" is in the specification, too often candidates recorded it as a synonym for "Normal Reaction" and answered accordingly.

Question No. 7

(i) Frequently this part of the question was answered correctly, with candidates integrating the acceleration of *A*.

(ii) This part of the question caused a large number of circular solutions to be presented (which gained no marks). Candidates could (and did) deduce that $U = 3$ from the position of *B* when $t = 5$.
The substitution of $U = 3$ and $t = 5$ into the distance formula of *B* was then held to verify the value of 25 m. That said, the correct solution based on differentiation of the distance formula of *B* was frequently seen.

(iii) There were a lot of very good answers to this demand. Candidates found the correct strategy for finding the final velocity of *B* and executed it precisely.

4729 Mechanics 2

General Comments:

The general report by examiners suggests that the response to this demanding paper was particularly good with very few candidates not up to the task. It was noticeable that there were many candidates who applied the principles of mechanics well enough but then didn't have the level of algebraic manipulation required to finish the job, particularly in questions 4, 5 and 6.

A recurring message in the report for this module is that candidates would be best served by drawing a force diagram, with all forces indicated, to aid their solutions to demands. Also candidates are reminded that when a request for a direction is made, then a clear unambiguous statement is required with a possible diagram also.

Comments on Individual Questions:

Question No. 1

(i) Most candidates scored full marks for this part.

(ii) Again most candidates scored full marks, with only a minority making an error in the weight component.

Question No. 2

(i) Most candidates made a good effort on this part with the only significant error seen was in how to deal with the distance of 4 m, in which some candidates read this as being the vertical height rather than a distance along the slope.

(ii) By far the most common approach to this question by candidates was an energy method, which in most cases had all energy terms present but not always with the correct sign. The candidates who used a Newton's 2nd Law approach usually answered the question correctly with an insignificant few failing to have all forces in their equation of motion.

Question No. 3

(i) Although this is a standard question, significant errors were made in an attempt to show the given answer. The key word "lamina" was often missed by candidates, who proceeded to use the centre of mass of an arc rather than a sector. It was also common to see a candidate thinking the shape was a framework. Even more concerning was the number of candidates who used the area of full circles in their calculation, which did lead to the given answer so that they were blissfully unaware of the error that had been made.

(ii) Candidates are improving in indicating the point about which they are taking moments and the most obvious place was about point *A*. However some used *AB* as 3*a* in their working instead of 6*a*, as well as others using $\sin 40^{\circ}$ in an attempt to find the distance associated with the tension. Some chose to take moments about other points, the most common being O, but proved to be less successful in answering the question. The lack of a full force diagram meant that moments of the force(s) at the pivot were omitted.

(iii) A good attempt was made at this with candidates gaining credit if they had a wrong tension in part (ii). However a significant number were not awarded the final mark due to the lack of clarity of the direction. Candidates must be aware that 79.1° to the horizontal is insufficient as this could mean either above or below the horizontal. A diagram would have helped to clarify the situation.

Question No. 4

(i) The usual method of resolving parallel to and perpendicular to the direction of the acceleration was successfully employed by the majority of candidates. However some used the wrong radius in their acceleration and others found the algebraic manipulation to achieve the given answer a challenge. A minority of candidates attempted to resolve in other directions with not much success, failing to realise that the acceleration has a component in the direction they were resolving.

(ii) Most candidates were aware that using $R = 0$ was the way to proceed in this question. However two requests were made with a significant number not attempting to find the tension in the string. In other cases, candidates only attempted the angular speed despite the request being for the speed of the particle.

Question No. 5

(i) A good diagram showing all forces acting on the ladder was beneficial to those who included it. Taking moments about A was the most common method to find the friction. However this method does initially lead to the normal reaction at the wall being the given answer. Some went no further thinking they had shown what was required. Those who took moments about B were less successful, mainly due to at least the moment of one force missing, typically the normal reaction at A.

(ii) Many candidates made a reasonable attempt to use $F = \mu R$ after finding the normal reaction on at B, but many couldn't solve this to get x.

(iii) This proved to be the most difficult request on the paper. Of those who made any attempt, many, wrongly, assumed that the friction found in part (i) still applied even though this was a completely new scenario. A significant number of candidates did not even attempt the question.

Question No. 6

(i) A good source of marks for many candidates. A few made sign errors in the two equations but in most cases both the linear and quadratic equation were initially correct. The manipulation to obtain a quadratic in a single v often left much to be desired – errors when dealing with the fractions were abundant and several candidates thought that since $4v_A = 2 - 3v_B$, $4v_A^2$ could be replaced by $(2 – 3v_B)^2$. The necessity for answers to be positive was often not recognised, directions were sometimes indicated by arrows rather than a statement and statements, when given, more often than not included reference to left/right, or moving apart rather than the more explicit 'reverse direction to original'.

(ii) The majority of candidates, who found a solution to part (i), were able to use the restitution equation, with only a few sign errors seen due to not taking into account the vector nature of the values used.

Question No. 7

(i) As reported last year, this question had two demands of which some only responded to one of them, usually the height of C above the ground. This height was correct for the majority of candidates. Of those who went on to find the distance AB, most candidates were equally successful.

(ii) The common mark for this was 3, due to the inadequate or non-existent indication of the direction of the impulse. Even so a significant number of candidates subtracted the two positive impulses they had found.

(iii) There were a lot of very good answers to this demand. While most candidates knew to use restitution, not all got it right – but many of these went on to score 6 marks. Similarly, many of those not using restitution at the start went on to gain the 4 marks allowed by the mark scheme. A small number of candidates found the horizontal distance of *Q* from *C* only, appearing to believe that the question had been answered.

4730 Mechanics 3

General Comments:

This paper proved challenging for the better candidates, with very few gaining more than about 65 of the 72 marks available. Weaker candidates were generally able to make some progress on each of the questions. A small number of candidates failed to complete the question paper, but in these cases there was often an indication of excessive time spent on one of the earlier questions.

Most candidates presented their work well, with clear and understandable working. Candidates who made errors and had to have a second attempt at a question usually made appropriate use of additional paper.

Some candidates did not answer questions fully. For example, in question 3(i) some candidates omitted to find the component of the velocity of *B* after they had found the speed of *A*, and in question 6(ii) some candidates did not attempt to find the component of the force acting on *AC* at *C* even though they had correctly arrived at the given answer for *R*. A similar issue arose in question 5(i) where some candidates found the angle *θ*, or cos *θ*, but failed to find the height of *P* above its initial level.

Comments on Individual Questions:

Question No. 1
(i) Most candidates drew a correct diagram for this situation and used the cosine rule to find the magnitude of the impulse. Those who then used the sine rule to find the angle usually failed to realise that the interior angle of the triangle was obtuse. Candidates who found components of the impulse, or used the cosine rule to find the angle, did not have any difficulty here. A small number of candidates did not get the momentum (or velocity) triangle correct.

(ii) Most candidates either had this correct, or correct on follow through from part **(i)**. There were some, however, who had quite different answers for both the magnitude of the second impulse and its direction.

Question No. 2
(i) Most candidates were able to use Newton's second law correctly and to integrate the equation they found. A small number of candidates omitted to give the expression for the velocity explicitly. Rather more candidates had trouble with the minimum value of the velocity, with $4\,\mathrm{m\,s^{-1}}$ and $0\,\mathrm{m\,s^{-1}}$ being seen on some scripts.

(ii) Most candidates did this well. Those with a small error in velocity in part **(i)** were able to gain some credit. A small number of candidates did not understand that they had to find the distance travelled in order to find the average velocity.

Question No. 3
(i) Many candidates gained full marks for this part, although some made a sign slip in one or other of the equations, some made an error in one or both of the masses and others made an error in solving the simultaneous equation. A small number of candidates failed to find the speed of *A* after finding the component of its velocity along the line of centres and a small number of candidates omitted to find the component of the velocity of *B* along the line of centres.

(ii) While there were many good answers, a considerable number of candidates lost a mark by failing to point out that the speeds of *A* and *B* perpendicular to the line of centres were equal.

Question No. 4
The most common way of tackling this question was to find the equilibrium position for the system and the elastic energy in each string there. The maximum kinetic energy of *B* is the difference between and the elastic energy in the stretched string at the start, and the sum of the energy in each string at the equilibrium position. Many candidates did this successfully; though some candidates defining a variable other than the extension of one of the strings had difficulties. Other candidates used the fact that the motion of *B* would be simple harmonic, found the amplitude of the motion and then the greatest kinetic energy of *B*. Candidates using this method sometimes had a little difficulty dealing with the mass of *B* (which cancels out) and sometimes gave the greatest speed (in terms of mass) instead of what was required. A small number of candidates used the first method to find the amplitude of the motion, and then did the whole question by the SHM method, duplicating their earlier work; even so, these candidates were often completely successful. Another method was to find an expression for the kinetic energy of *B* in terms of an unknown length, and use differentiation or completing the square to find the answer.

Question No. 5
(i)   This part proved straightforward for most candidates, though some made sign errors and some failed to include the component of the weight term when finding an expression for *T*. A small number of candidates left the answer as θ, or $\cos \theta$, failing to find the height required.

(ii)   Many candidates tackled this by finding the tension in terms of *U* for a general position, and then putting it equal to zero. However, the majority using this method failed to progress any further, and those that did almost always only found the greatest possible value of *U* for the string to not remain taut in the subsequent motion. Similarly, other candidates who looked at the specific position where the string went from being taut to not being taut (or vice versa) almost only considered the topmost point on the motion and ignored the fact that the string will always be taut if the angle it makes with the upward vertical is more than 90°.

Question No. 6
(i)   Almost all candidates followed the instructions and found a correct equation relating *W*, *R* and *F*. A small number of candidates made errors confusing sine and cosine, and some made slips with signs.

(ii)   This part could be tackled in a number of ways and most, but not all, candidates attempted to find an appropriate second equation to solve along with their answer to part **(i)**. The most common second equation was found by taking moments about *A* for the equilibrium of rod *AC*, and candidates then had to use the fact that the friction forces at *B* and *C* were equal together with the fact that the sum of the normal reactions and *B* and *C* was 3W. The next most common second equation was to take moments about *C* for the equilibrium of the whole system; quite a number of candidates attempting this method had the friction force at *B* acting in the wrong direction, while others omitted this term. Another method was to take moments about *C* for the equilibrium of *AC*; this required candidates to use the forces at *A*, and many did this quite successfully. Some candidates omitted to find the vertical component of the force acting on *AC* at *C*; a small number of candidates, having failed to show the given result, assumed the value of *R* and then found the vertical component of the force acting on *AC* at *C*.

(iii)   While there were many correct, efficient solutions to this part, not all candidates realised that rod *AB* would be the one on the point of slipping. Some candidates got over this problem by finding the possible values of the coefficient of friction for both *B* and then *C* slipping – and most of these then gained full marks by then indicating rod *AB*.

Question No. 7
(i)   This part was done correctly by the great majority of candidates.

(ii)   Most candidates were able to do this correctly. A common error, however, was to assume that the potential energy lost by Q while falling was equal to the kinetic energy of the combined particle after they had coalesced. Many such candidates still claimed to have established the speed of $2.1\,\mathrm{m\,s}^{-1}$.

(iii)   Many candidates were able to find the centre of the motion, either from their attempt at showing simple harmonic motion or by considering the forces acting if the particle was at rest in that position. While most candidates were able to use Newton's second law to attempt to establish simple harmonic motion, only a minority did so correctly. The main error was to use a mass of m, rather than of $4m$, either once or twice in their equation. Partial credit was given to candidates attempting to use a correct form of equation to find the amplitude, but only those with a correct value of ω were able to get the correct value. A small number of candidates attempted to find the amplitude by considering energy, but most of these candidates failed to include potential energy in their calculations.

(iv)   Some candidates failed to attempt this part, and most who made an attempt did not consider all parts of the motion. This part was largely marked on the method used rather than accuracy, allowing 'follow through' from whatever values the candidates had for amplitude and ω. Most candidates who attempted the part were able to find the time it took Q to fall to the point where it coalesced with P. Candidates were next expected to find the time taken from this point to the centre of the simple harmonic motion. After this candidates had to realise that the combined particle performed simple harmonic motion only while the string was extended, so they needed to find the time to the point where the string next became slack, the speed of the particle at that point and then the time from there to the top of the motion. Fully correct solutions to this challenging question were rarely seen.

4731 Mechanics 4

General Comments:

The work on this unit was generally of a very high standard. Many of the candidates were very competent and demonstrated a sound understanding of the principles of mechanics covered in this unit. However, a small number of candidates struggled with the majority of the paper and were not able to apply principles appropriate to the situations. Candidates seemed to be particularly confident when using calculus to find the x - coordinate of the centre of mass of a lamina, applying the principle of conservation of mechanical energy and using energy to investigate stability of equilibrium. Topics which were found more challenging included relative velocity, finding the y - coordinate of the centre of mass of a lamina and applying the rotational form of Newton's second law. Candidates appeared to have sufficient time to complete the paper. The standards of presentation and communication were high, though some candidates failed to include necessary detail when establishing given answers.

Comments on Individual Questions:

Question No. 1
The majority of candidates correctly calculated the moment of inertia of the square lamina about an axis passing through its centre and then either correctly applied $C = I\alpha$ together with $\omega^2 = \omega_0^2 + 2\alpha\theta$ or correctly applied the work-energy principle to find ω. The most common errors were in the calculation of the moment of inertia of the lamina with a number of candidates using 0.2 rather than 0.1 in the formula $\frac{1}{3}m\left(a^2 + b^2\right)$ or incorrectly finding the moment of inertia about a vertex rather than the centre of the square lamina.

Question No. 2
Relative velocity remains a difficult and challenging topic for many and a number of candidates left all three parts of this question blank. However, there were a significant number of candidates who answered all three parts of this question correctly. The most succinct and efficient solutions in part (i) were from those candidates who applied both the cosine and sine rules. In this part the most common error was using an incorrect angle and it was noticeable that a number of candidates incorrect summed 30 and 35 as 75.

Parts (ii) and (iii) were also answered well with the majority of candidates applying the correct method for finding the shortest distance in part (ii) and the time taken in part (iii). A significant number, however, used the cosine of the correct angle rather than the sine in part (ii) and a number of candidates used the distance from part (ii) in their attempt to work out the time to closest approach in part (iii).

Question No. 3
The vast majority of candidates correctly showed the given result for the gravitational potential energy of the system in part (i). Nearly all candidates correctly calculated the length of *BD* as $2a\cos\theta$ although some spent a considerable amount of time using, and simplifying, their length for *BD* (after applying the cosine rule) rather than exploiting the fact that triangle *ABD* was isosceles. Those that did have the correct length for *BD* nearly always went on to correctly find the elastic potential energy of the string. A number of candidates struggled with finding the gravitational potential energy of the two rods *AB* and *BC* with a number not showing sufficient detail of how they arrived at the given answer. While nearly all candidates correctly understood the method for calculating the exact value of λ in part (ii) for some, however, the differentiation

of V required in this part was far too demanding. Many candidates struggled to calculate $\dfrac{d}{d\theta}\left[(2\cos\theta-1)^2\right]$ accurately with many omitting the negative sign or failing to include an additional factor of 2.

In part (iii) most candidates scored both method marks for attempting to differentiate their $\dfrac{dV}{d\theta}$ and substituting $\dfrac{1}{4}\pi$ for θ but very few correctly had the value of the second derivative as $mga\left(1+3\sqrt{2}\right)$ and hence could accurately determine that the equilibrium position was stable.

Question No. 4
In part (i) a small minority of candidates attempted to find the value of the x - coordinate of the correct lamina or instead found the y - coordinate of the uniform lamina given in part (ii). However, the majority of candidates correctly found the area of the lamina in part (i) but many did not go and correctly evaluate the integral $\dfrac{1}{2}\int y^2 dx$ with the most common error being to forget the factor of a $\dfrac{1}{2}$. While the vast majority of candidates attempted to use strips parallel to the y - axis in their attempt to find the required centre of mass a number attempted to use strips parallel to the x - axis but none were successful due to the difficulty in dealing with the corresponding geometry of the problem. Many did not appreciate that the centre of mass in this case would be given by $\overline{y} = \dfrac{\displaystyle\int_2^{2e} y\left(2-2\ln\left(\dfrac{y}{2}\right)\right)dy + 4}{4e-4}$ as (in this case) the horizontal strips are formed from the line $x=2$ rather than the y - axis.

Part (ii) was answered extremely well with many obtaining the correct exact value for the x - coordinate of the centre of mass of the lamina. Many dealt correctly with the required integration by parts for the integral $\displaystyle\int_0^2 2x e^{\frac{1}{2}x} dx$ but it was surprising how many candidates used integration to work out the centre of mass of the triangular part of the uniform lamina when it was expected that candidates would simply state that the centre of mass (of this triangular lamina) was at a horizontal distance of 3 units from the y - axis.

Question No. 5
Part (i) was answered extremely well with nearly all candidates correctly deriving the given answer for the moment of inertia of the rod about the stated axis.

In part (ii) many candidates appreciated the need to apply the conservation of angular momentum to find the initial angular speed of Q although some incorrectly believed that the principle of conservation of energy could be applied to a situation in which a collision between two bodies had occurred. Regardless of the method employed nearly all candidates correctly calculating the required moment of inertia of Q.
Part (iii) was tackled with varying degree of success as although nearly all candidates (who attempted this part) appreciated the need to apply the conservation of energy but many did not calculate the change in potential of energy of Q correctly. The calculation for the change in kinetic energy was far more successful.

Part (iv) was often left blank but a significant number of candidates appreciated that Q would make complete revolutions provided that when $\theta = \pi$, $\left(\dfrac{d\theta}{dt}\right)^2 > 0$. A number of candidates incorrectly stated that $v^2 \geq \dfrac{160}{3}ga$ or only stated the set of values for v without ever stating the corresponding values for v^2.

Candidates found part (v) demanding and only a few succeeded in getting R correct. In this part only a minority of candidates derived the correct equation of motion involving the transverse component of the acceleration. The most common errors included sign errors, using a mass of m rather than $3m$ and using a radius of a or $4a$ rather than the correct $\dfrac{8}{3}a$.

Question No. 6
In part (i) the vast majority of candidates correctly derived the given result for the moment of inertia of the pendulum about the stated axis of rotation. However, for the case of the rod, a number of candidates began by stating that the moment of inertia of the rod about A was given by $\dfrac{4}{3}(3)\left(\dfrac{1}{2}\right)^2$. These candidates then went on to obtain an expression of $1 + 3x^2$ for the moment of inertia of the rod (about the given axis of rotation) therefore implying that the parallel axis theorem can be thought of as $I_A = I_B + m(AB)^2$ rather than the correct $I_A = I_G + m(AG)^2$.

The responses to part (ii) were mixed although many candidates correctly stated the centre of mass of the pendulum from the axis P as $\dfrac{19}{20} - x$ or correctly considered the moment of all three weights separately. However, most candidates then went on to apply the rotational form of Newton's second law incorrectly as many failed to realise that the component of the weight of the pendulum is in the opposite sense to which θ is increasing.

In part (iii) many candidates correctly applied the small angle approximation to their angular acceleration expression found in part (ii) but many incorrectly claimed that their equation of motion was of the form $\ddot{\theta} \approx -\omega^2\theta$ without realising that their expression for the angular acceleration was positive for small values of θ. Many candidates failed to explicitly state that the motion was approximately simple harmonic and many failed to give sufficient detail in calculating the given result for the approximate period of oscillations.

In part (iv) many candidates appreciated the need to use differentiation in an attempt to find the value of x for which the approximate period of oscillations is least but many over complicated matters by differentiating the entire expression for the period without realising that only the expression $\dfrac{20x^2 - 38x + 24}{19 - 20x}$ needed to be considered. It was disappointing at this level that a number of candidates failed to differentiate this quotient correctly although a good number did obtain a correct quadratic equation for x and hence the correct single value of 0.405 (correct to 3 significant figures) or the exact value of $\dfrac{19 - \sqrt{119}}{20}$.

CER Report to Centres: Probability and Statistics 4732/01, 4733/01, 4734/01, 4735/01

These general comments on the Statistics specifications are very similar to last year's. The overall standard of numerical work on these units remains pleasingly high. Less good are the responses to questions that require verbal answers; it is plain that many candidates are unable to answer verbal questions, particularly those that they have not seen before, except by trying to recite phrases familiar to them. Mark schemes are increasingly designed not to reward this type of response. Examiners are not looking for specific words but for an understanding of what the words mean. It is emphasised that it is not a good policy to attempt to memorise fixed words and phrases in order to answer verbal questions (especially where, as in the case of the modelling assumptions for the Poisson distribution listed in a designated textbook, the conditions are at best misleading and arguably wrong).

In general, questions on which the marks are low are not those that are intrinsically difficult but those that are unfamiliar or new. Candidates often seem to be trying to remember a question that they have seen before that resembles the one in front of them. This is not likely to be a successful approach and it suggests that learning and revision should not be limited to working through past papers.

It is pleasing to find that many Centres have responded to last year's request that any extra paper needed by candidates does not consist of 8-page or 12-booklets but if at all possible single sheets only. It is extremely unusual for 4-page continuation booklets to prove insufficient. Not only does the use of longer answer booklets waste paper, it adds substantially to the task of Examiners, as each blank page requires a separate acknowledgement. It would be appreciated if this message could again be conveyed to Examination Officers at Centres.

Candidates who write their answers in the wrong sections of the Printed Answer Book tend to cause havoc for themselves and for the Examiners. Candidates who write answers in the wrong place, or who have filled the given space, should always continue on separate sheets of paper.

The present specifications do not require candidates to use calculators that find, for example, cumulative Poisson probabilities or inverse normal distribution values, and therefore questions are set and marked in such a way as to confer no disadvantage on candidates who do not have them. Candidates *with* such calculators are therefore reminded that they are at considerable risk of losing a lot of marks if they fail to write down the working expected. For instance, candidates who get nearly but not quite the correct answer, or even a common wrong answer, without working can only be awarded zero marks. This year many candidates lost marks in this way. When a question specifies that a formula is to be used or working is to be seen, candidates who obtain the answer directly from their calculators will score no marks. Likewise, the use of calculator notation such as cdfbinomial, or the comment "using GDC" does not qualify for method marks, although these can be given subsequently if the right answer is obtained.

Centres are encouraged to teach candidates the correct use of the formulae in MF1. More time devoted to this might also reduce the number of instances of weak candidates desperately trying to find a relevant-looking formula and quoting one that is completely useless for their purposes. A common instance of this is candidates who quote the PDF of the normal distribution,

$\dfrac{1}{\sigma\sqrt{2\pi}}\,e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, which is rarely of any use at all.

4732 Probability & Statistics 1

General Comments

Candidates generally found this paper reasonably accessible. Most candidates scored well on the standard calculations such as those in questions 1(i), 2(i)(a) and 5(i)(a, b &c). A few questions contained relatively non-standard requests (eg 1(ii), 2(ii)(a and b), 4(iii), 5(iii) and 7(iii)(a & b)) and some candidates could not handle the slightly different approaches that were needed. In particular, in questions 1(ii), 5(iii) and 7(iii)(b) many candidate were unable to identify all the relevant cases and many were unsure when to add, and when to multiply, probabilities. Most candidates were surprisingly weak in the combination question, 6(a), but scored fairly well on 6(b)(i and ii).

Answers given in words

The questions that required answers given in words (2(i)(b) and 7(i)) could be answered correctly using absolutely standard responses, learnt by rote. Even so, these were poorly answered by many candidates. In particular, question 7(i) produced many incorrect answers, obviously learnt by rote. In both questions 2(i)(b) and 7(i), many candidates showed no understanding either of the context or of the principles involved.

Rounding

Centres should note the rubric about giving answers correct to three significant figures. A few candidates lost marks by premature rounding or by giving their answer to fewer than three significant figures without having previously given an exact or a longer version of their answer. It is important to note that although an intermediate answer may be rounded to three significant figures, this rounded version should not be used in subsequent working. The safest approach is to use exact figures (in fraction form) or to keep intermediate answers correct to several more significant figures.

Two errors in rounding that occur frequently are the following. If the third significant figure is zero, candidates often omit it. And some candidates think that, for example, 0.92 is actually three significant figures, the "0" being the first significant figure.

Candidates who give alternative solutions

More frequently than usual, candidates gave two solutions to a particular question and did not indicate which solution they wished to be marked. Examiners are <u>not</u> required to mark both solutions and choose the best one. They are required to mark just <u>one</u> of the given solutions. Centres should emphasize to candidates that they must make a choice between their attempts and should cross out the solution that is not to be marked.

Drawing

In question 4(ii) a few candidates' drawings were rather faint. Others included rubbings out that became visible to the marker on the computer screen. Candidates should be made aware that their answers will be scanned and read on a computer screen and therefore some clarity may be lost, unless they draw clearly and rub out mistakes thoroughly.

Use of statistical formulae and tables

The list of formulae, MF1, was useful in questions 2(i)(a), 4(iii) and 5(i)(a) (for binomial tables). Candidates generally used the MF1 booklet well. In question 2(i)(a) very few candidates quoted

their own (incorrect) formulae for r rather than using the one from MF1. A small number of candidates incorrectly thought that $S_{xy} = \Sigma xy$ or $\Sigma x^2 = (\Sigma x)^2$. In question 4(iii), Σd^2 was sometimes misinterpreted as $(\Sigma|d|)^2$ or even $(\Sigma d^2)^2$ and the formula was sometimes misquoted

as $\dfrac{6 \times \Sigma d^2}{n(n^2-1)}$ or $\dfrac{1-6 \times \Sigma d^2}{n(n^2-1)}$ or $1 - \dfrac{6 \times \Sigma d^2}{n}$ or $1 - \dfrac{6 \times \Sigma d^2}{n^2(n-1)}$, despite the formula being given clearly in MF1.

In question 5(i)(a), many candidates used the formula rather than the table, which is quite understandable, but leads to a somewhat longer method than necessary. In question 5(i)(b) some candidates' use of the binomial tables showed that they understood the entries to be individual, rather than cumulative, probabilities.
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Use of calculator functions

Increasingly nowadays, calculators can provide answers using statistical functions, binomial functions etc etc, without the need to quote a formula and substitute values into it. The problem here is that if candidates write down their answer with no working, they can only score either full marks or no marks, with no possibility of gaining any credit for partially correct working. In most cases, the use of such functions saves very little time and it is advisable to show working instead. However, if candidates wish to use these functions, they should input all the relevant data twice in order to check their answer.

It should also be noted that, without working, even a correct answer is not guaranteed to gain full marks.

Other points

Few candidates appeared to run out of time.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Some candidates ran out of space and continued on the back page, or in a separate answer booklet. This is obviously quite acceptable, but centres should emphasise the need for candidates to give a clear indication of the fact that they have written further working on another page.

Comments on Individual Questions

1)(i)   E(X) presented no difficulty to most candidates, although a few divided their answer by 4. Some candidates failed to subtract $(E(X))^2$ from $\Sigma x^2 p$ when finding Var(X).

(ii)   Some candidates considered the cases where X took the value 2 twice but omitted the case where X takes the value 2 three times. Others considered, for example, "2, 2 and Not 2" but did not take account of the fact that there are three possible orders. Some candidates considered all the separate cases (1, 2, 2), (2, 2, 2), (3, 2, 2) and (4, 2, 2) which is considerably longer than necessary and therefore gives more opportunity for arithmetical errors. Some used the complement method, presumably prompted by the words "at least", although in this case it is longer than the direct method.

2)(i)(a)The vast majority of candidates answered this well, with the actual calculation being carried out accurately in most cases. A few truncated their answer to 3 significant figures, rather than rounding it, thereby losing a mark. There were very few cases of premature rounding of one or more S-values.

(i)(b) This is a simple, standard, and very common, question. Nevertheless, many candidates appeared not to be aware of the fact that correlation does not imply causation.

(ii)(a) Disappointingly few candidates gave the simple answer that the gradient of the regression line is positive and therefore the output increases. The majority actually calculated some of the values of the output. Some even calculated all 12 values! Many gave only the first two or three values, but this did not gain the mark because it does not comprise sufficient evidence that the output <u>generally</u> increased.

In parts (ii)(b) and (ii)(c) many candidates used the unnecessarily long methods described below. They gained full marks if they obtained the correct answers, but the large amount of arithmetic involved meant that they were liable to make arithmetical errors.

(ii)(b) Most candidates found $\bar{n}$ but, again disappointingly, many found $\bar{z}$ by using the equation of the regression line to find all twelve values of z rather than by substituting their value of $\bar{n}$.

(ii)(c) Again, many candidates worked out all 12 values of the output and averaged them, rather than multiplying their answer to part (ii)(b) by 12.

3)(i) Most candidates found the mean of m correctly, although a large minority only found the mean of $(m - 150)$. The variance caused problems for many candidates. Only a minority appreciated that they only needed to find the variance of

$(m - 150)$. Many tried to "uncode" the data, for example by finding $\frac{1768}{52} + 150$ (or even

$\frac{1768}{52} + 150^2$) before subtracting either the (mean of m)^2 or (the mean of

$(m - 150))^2$. Some candidates mixed coded and uncoded values, giving $\frac{1768}{52} - "146.5"^2$ or

$1768 - ('-3.5^2)$. Some attempted firstly to find Σm^2. But most of these candidates appeared not to understand the meaning of the "Σ" sign and made little or no progress. Some candidates gave $\Sigma(m-150)^2 = 1768$, which is correct, but continued with working such as $\Sigma m^2 - 300m + 150^2 = 1768$. A strange, but not uncommon, error in the variance calculation was $1768 - \frac{(-3.5)^2}{52}$

(ii) Only a few candidates made any progress in this part. Some tried to find Σm^2 from

$\Sigma(m - 150)^2$, but most did not know how to handle the Σ sign.

(iii) Many candidates saw the point and recognised that 140 is the lower quartile. From that point, most candidates just divided 52 by 4 to give an answer of 13. These candidates gained full marks. However, the correct method involves finding $\frac{52+1}{4}$ or $\frac{26+1}{2}$, which leads to the conclusion that the lower quartile lies between the 13th and 14th values and hence there are 13 values below the lower quartile. Many candidates used a wholly incorrect method using "scaling", such as $\frac{52}{46} \times 10 = 11$.

(iv) Many candidates answered this question correctly. However, some misread the description of outliers given in the question and found, for example, 1.5 × the upper quartile. Many other candidates just gave a verbal answer with little or no calculation to support it. These generally gained no marks or possibly just one mark. A few candidates quoted the convention that outliers are indicated by dots on a box-and-whisker plot. The wording of this question meant that these candidates could not score more than one mark.

4)(i)   Most candidates gave the correct answer, although a few gave answers such as "1" or

"-0.9".

(ii)   This question seemed to catch many candidates unawares and many failed to see the point about the difference between the two coefficients. Some gave five points in a straight line. Some gave apparently randomly placed points having a general tendency towards positive linear correlation, but with y not always increasing with x. A few appeared to be trying to give a correct answer, but inadvertently placed one point vertically above another, or horizontally on the same level as another.

(iii)   Faced with no values of ranks, a large minority of candidates were unable to start this question, not realising that the ranks can be read off the diagram. A few candidates postulated that $r_s = 1$, then deduced that $\Sigma d^2 = 20$, and substituted this into the formula and, predictably, obtained the answer $r_s = 1$. Some candidates attempted to read off coordinates, which is quite acceptable so long as the consequent values are ranked. But some candidates just used their raw coordinates to find Σd^2 without first assigning ranks. A few candidates read off the ranks incorrectly and some ranked the x-values and y-values in opposite directions. Some candidates gave a value for Σd^2 without showing any ranks. Many candidates found either $\Sigma d = 2$ or $\Sigma d^2 = 2$, and then squared the "2" before substituting in the formula for r_s.

5)(i)(a)This question was answered correctly by many candidates. A few read the wrong value from the table. Some used the formula rather than the tables, which took considerable time and gave much scope for arithmetical errors.

(i)(b)   This question was answered correctly by many candidates. A few read the wrong values from the table. Those who used the formula were largely successful.

(i)(c)   Most candidates recognised that the relevant formula is npq and gave the correct answer. However, a few attempted to start from first principles, using

$\Sigma x^2 p - (\Sigma xp)^2$. None of these candidates understood the formula they were attempting to use and none gained any marks.

(ii)   The most common error was the omission of the expression for $P(Y = y)$. The cause for this may have been ignorance of what the question meant, or it may have been that candidates simply did not read the question carefully enough.

(iii)   This question proved too difficult for most candidates. Many found some of

$P(Z = 0)$, $P(Z = 1)$ and $P(Z = 2)$, but did not proceed to square the values they had found. Some doubled, rather than squared, their probabilities. Others found only two of these probabilities but then squared and added them correctly.

6)(a)   Most candidates started correctly with $^{12}C_5$, but many then proceeded to find $^{12}C_4$ and $^{12}C_3$ and either added or multiplied these. A few found the correct three combinations, but then added them, instead of multiplying. Many candidates attempted to find a probability, with denominators such as $^{12}C_5$, $^5C_3$ or $^4C_3$

(b)(i)   This was answered correctly by many, but some just found 7! Others seemed unaware of the standard method and used their own ingenuity, finding, for example, 7^7.

(b)(ii) The fact that the answer to this question is simply the reciprocal of the answer to the previous part was missed by a large number of candidates. Many started from scratch, using either combinations or fractions, and some of these candidates were successful. Others used the answer to part 6(b)(i) as the denominator, with numerators such as $2! \times 3! \times 2!$.

(b)(iii) Most candidates did not use the most straightforward method, which is simply the product of four fractions. (The fact that a probability method may be simpler than a method using permutations and/or combinations, is one which could do with more emphasis.) A few candidates used the fractions method but then incorrectly multiplied the result by, for example, $^7C_4$, or divided by, for example 4!. Many attempted to use permutations (which is much better, in this case, than using combinations), often successfully. Some found the correct denominator of $^7P_4$ but gave a numerator of $^3P_2$, omitting to multiply by $^2P_2$. (Perhaps they mistakenly thought that $^2P_2 = 1$, getting confused with the fact that $^2C_2 = 1$). Some used a denominator of 7! instead of $^7P_4$. Those who attempted (unwisely) to use combinations were rarely successful in scoring even one mark. Many gave a denominator of $^7C_4$ but with an incorrect numerator. A few correctly found $\frac{^3C_2 \times {^2C_2}}{^7C_4}$, but failed to take account of the fact that this included the same arrangement several times. The necessary multiplication by $\frac{1}{(\frac{4!}{2!2!})}$ or $\frac{4}{^4P_4}$ or $\frac{1}{6}$ was given by only a tiny minority of those candidates who chose to use combinations. Some multiplied $\frac{^3C_2 \times {^2C_2}}{^7C_4}$ by 6, instead of by $\frac{1}{6}$.

7) Throughout this question, some candidates used formulae for probabilities in a geometric distribution. This leads to rather clumsy expressions such as

P($X = 2$) = $0.2 \times 0.8^{(3-1)}$ and P($X < 10$) = $1 - (1 - 0.2)^{(10-1)}$. In fact a "common sense" approach to questions on the geometric distribution, without recourse to formulae, is quite feasible and possibly simpler. For example, $X < 10$ means that the first success occurs before the 10th trial. In other words it is not true that the first 9 trials are all failures, hence P($X < 10$) = $1 - 0.8^9$.

Binomial coefficients appeared occasionally in answers to this question.

(i) Very many candidates appeared not to understand what is meant by "conditions". Some quoted conditions taken from text books. This led to two errors. Firstly many candidates omitted the context. For example "The probability of success must be constant for all trials". Without context, no marks could be gained. Secondly many candidates quoted, as conditions, facts about the situation that are implicit in the question. For example "There must be only two outcomes to each shot." or "She must continue her attempts until she scores her first goal." Some candidates referred to probabilities being independent, rather than shots.

(ii)(a) Most candidates answered this question successfully

(ii)(b) The most common error in this question was $1 - 0.8^{10}$. Another error was to include a 0.2 in the calculation, for example $1 - 0.2 \times 0.8^9$ or just 0.2×0.8^8. Some candidates omitted the "1 -", finding P($X \geq 10$) instead of P($X < 10$). Other candidates used the "long method", finding the sum of a large number of probabilities. Some of these candidates omitted one probability or added an extra one or made arithmetical errors. A few candidates used the long method but found the sum of the relevant terms using the formula for the sum of a geometric progression.

(ii)(c) This was found to be difficult by many candidates. Some candidates had a basically correct method, but with an incorrect index, such as $0.8^9 - 0.8^{20}$. Some subtracted the wrong way round and just chose to ignore the resulting minus sign. Others multiplied instead of subtracting. Many candidates showed a basic misunderstanding of how to handle a range of values between two limits. These gave calculations such as

0.8^9 - $(1 - 0.8^{19})$ or even $0.8^9 \times (1 - 0.8^{19})$. Another error was to include 0.2 in the calculation, for example 0.2×0.8^9 - 0.2×0.8^{19}. Of course, many candidates used the "long method" and a good number were successful, although some omitted one probability or added an extra one or made arithmetical errors. A few candidates used the long method but found the sum of the relevant terms using the formula for the sum of a geometric progression.

(iii)(a) There was much evidence of muddle in candidates' responses to this question. A few candidates realised that the result of Nadine's first shot was irrelevant and gave the elegant method $0.2^2 = 0.04$. Many used only one of the two possible routes, for example $0.2 \times 0.3 \times 0.2$. Others saw that there were two routes and many of these candidates were successful. A common error was to ignore the words "The winner is the first one to score two goals", treating this like a geometric distribution, with Marie having to miss on her first shot and score on her second, eg $0.8 \times 0.7 \times 0.2$ or just 0.8×0.2.

(iii)(b) This was answered well by a few candidates who correctly identified the three possible routes. Some included only two of the routes, eg

$0.2 \times 0.3 \times 0.8 \times 0.2 + 0.8 \times 0.3 \times 0.2$, and others only one. For example some candidates assumed that Marie has to fail on both her shots, thus

$0.8 \times 0.3 \times 0.8 \times 0.3$. A common error was to ignore Marie and just find

0.3×0.3.

A few candidates used the very elegant method $(1 - 0.2^2) \times 0.3^2$.

4733 Probability & Statistics 2

General Comments:

This was found by many to be a fairly straightforward paper and much excellent work was seen; marks were correspondingly high. As noted last year, conclusions to hypothesis tests are now very often well stated, although it is again emphasised that it is always wrong to conclude that there is evidence that the null hypothesis is true. The correct statement requires a double negative, for example "there is insufficient evidence that the mean number of failures has been reduced".

The verbal answers as usual revealed specific misunderstandings. As usual many candidates seem to be unable or unwilling to try to think things out for themselves but rely on answers that they have learnt from previous mark schemes, often parrot-fashion. This is undoubtedly not good education practice and mark schemes are designed not to reward such answers.

As in the past it is plain that a substantial number of candidates think that in order to use a Poisson distribution the number of events occurring in a fixed interval must actually be constant. It is hard to see how this belief is compatible with any understanding of probability distributions.

As last year, there were many candidates who, in answering hypothesis tests based in the binomial or Poisson distributions, gave a method that could not be clearly identified, apparently half way between the critical region method and the probability method. This is hard to credit appropriately as well as being poor practice. In particular, if the critical region method is used, it is necessary to state the critical region unambiguously. "Critical region is ≥ 19" is needed, and not just, for example, "critical value = 19". Further, when stating the critical region candidates are expected to validate their answer by quoting the relevant probability.

The Central Limit Theorem remains a part of the specification that causes difficulties for many candidates; this is discussed in detail in Q8(iii) below.

Candidates with calculators that provide probabilities directly from standard distributions are reminded that failure to use the proper notation is a high-risk strategy. A candidate who writes, for instance "cdfnorm(35.5, 30, 5.477)", or "by GDC" can get full marks if the answer is completely correct but loses <u>all</u> relevant method marks as well as accuracy marks if the answer is wrong, even if that answer is recognisable as a standard mistake such as a missing continuity correction. Examinations aim to encourage, among other things, the use of correct mathematical notation, and calculator syntax of this type is not accepted as correct notation – for the present, anyway.

Comments on Individual Questions:

Question No.

Q.1   Almost everyone found this a very straightforward start.
        268: 147 (3 sf).

Q.2   There were many fully correct answer to this question, with only a few making the usual mistakes such as sign errors, use of σ^2 instead of σ, or 0.05 instead of 1.645.
        13.6(5)%.

Q.3   Again this question was done well. The proportion of candidates using the correct continuity correction is now very pleasingly high. Those who quote the conditions for the normal approximation to binomial in the form of inequalities must be sure to give the numerical values of *np* and *nq*; here it was essential to see, for example, "14 > 5". As usual, some candidates gave the second condition in terms of *npq*, which is wrong. 0.932.

Q.4   Again this question was very often correctly answered. There was a pleasing confidence in handling the algebra and arithmetic of the Poisson formulae; only a few candidates over-complicated, for instance by trying to take logs (usually wrongly). 5;   0.175.

Q.5(i)   A standard hypothesis test for a binomial parameter. The proportion of candidates who considered the wrong tail, or no tail at all, seemed lower than in the past, which is pleasing. To make the point clearly: with a sample value of 6 (and an expected value of 4.4) the probability that has to be found is $P(\geq 6)$, and not $P(> 6)$ or $P(= 6)$. The use of $P(< 6)$, although not wrong, should be discouraged, as comparison with large probabilities is not in the spirit of hypothesis testing. Often those who used the critical value method did not make it clear what the actual critical region was. "Critical value is 7" is not enough; it has to be "critical region is ≥ 7", and then "6 is not in the CR", or "6 < 7" has to be clearly stated. Candidates who did not state the critical region unambiguously risked losing the last two marks as well as earlier ones. $p = 0.2201 > 0.1$, do not reject.

Q.5(ii)   This verbal question revealed a lot of muddled thinking. The issue is whether the Head Students from the last 8 years can be taken as a representative of all Head Students from the period under discussion, and so the focus has to be on selecting the *years*. Many candidates instead attempted to apply standard binomial conditions to the way in which the Head Student was chosen (elected?) each year ("each Head Student must be chosen independently of the previous Head Student").

Q.6(i)   The standard Poisson conditions were trotted out by many candidates. As usual the conditions had to be given in context (hence not "events occurring" but, for example, "cars passing"). Also as usual, any suggestion of actually constant numbers ("cars must pass at a constant rate") did not gain credit, and the scenario was chosen with great care to make the often-quoted but almost invariably meaningless "singly" condition impossible (so that no reference to "singly" could score marks here). The mangled condition "average constant rate" was accepted on this occasion but might not be in the future; in any case Examiners suspected that many candidates were quoting it without really knowing what this condition means. The point is that considering all possible one-minute intervals (at the same time of day), the expected number of events is the same for each interval.

Q.6(ii)   Generally well done. The common wrong answer was 0.4491 from $P(\leq 7) – P(\leq 4)$. Those who did, for instance, $P(\geq 8) – P(\leq 3)$ scored no marks. 0.561.

Q.6(iii)(a) Again, generally very well done. If a numerical condition is quoted it has to be the one in the specification, namely $\lambda > 30$ (and some wrongly wrote "*n* > 30"). A few candidates rounded prematurely and obtained 0.159, forfeiting a mark. 0.158.

Q.6(iii)(b)   This was another challenging question. If on average a car passes every 2 seconds along a single track one-way road then it is likely that there is a steady procession of cars. The Poisson condition that this most obviously negates is that of randomness, though any sensible comment that negated independence was given credit. Those who focussed on "constant average rate" were rarely convincing; after all, the question stated that λ was 30, so there was no point in challenging this, and to say that the rate varies from one time of the day to another doesn't negate the use of Poisson for one of those periods in isolation.
Perhaps the fact that a likely scenario here really is, for once, "cars passing at constant rate" might help future candidates to appreciate the difference between "constant rate" and "constant average rate".

Q.7(i)   Many candidates have learnt that $\underline{x}$ is a value taken by the random variable $\underline{X}$, though some still seem to have no clear idea of what is happening and refer to "inputs into the probability function" or some such. If this question has focussed on such a widespread misunderstanding and helped to reduce it, it will have served a good purpose.

Q.7(ii)   Candidates made surprisingly heavy weather of this question (many scripts stretched to several additional pages), but it is simply the continuous equivalent of an often-asked S1 question. The intention was that candidates should use the given information to integrate the PDF between 2 and ∞ to obtain one equation, such as $\dfrac{a}{8} + \dfrac{b}{24} = \dfrac{3}{16}$, and use the result that the total probability is 1 to obtain a second equation such as $\dfrac{a}{2} + \dfrac{b}{3} = 1$. However, quite a lot of candidates didn't think of this second condition, although many used the equivalent method of finding the area between 1 and 2. Problems that arose included attempts to use 0 as a limit or thinking that P(> 2) means P(≥ 3).
As the question said "<u>show that</u> $\underline{a}$ = 1" it was not sufficient to produce two simultaneous equations and just write down the answers, perhaps from a calculator. Enough working was needed to get to the point where the value $\underline{a}$ = 1 was obvious, unless the candidate found b first. Some assumed that $\underline{a}$ was 1 and could get 3 marks out of 7 if they then obtained the correct value of $\underline{b}$ from a correct equation.
$\underline{b}$ = 1½.

Q.7(iii)   Many candidates obtained full marks on this question, and even if they had obtained the wrong value of $\underline{b}$ in part (ii) could score 2/3. There were pleasingly few mistakes with the algebra.
1.75.

Q.8(i) and (ii)   This was by far the least well answered question, and between them the two parts produced often produced chaotic results. The correct method was that part (i) was a straightforward test for the mean of a normal distribution, using the given variance with a divisor of 50, while in part (ii) it was necessary to multiply the given variance by 50/49 (and then divide by 50 again). Unfortunately a lot of candidates did not appreciate the difference between the two variances and attempted somewhat desperately to find some other difference between parts (i) and (ii), usually dividing by 50 in one part but not the other. Some wrote identical solutions to the two parts, which was at least honest! It may be worth spelling out that a divisor of n is always needed when the variable used for calculation is a sample mean.

Several candidates took 12.25 to be the standard deviation, which of course led to very wrong numerical answers (though they could still get most of the marks).

More pleasingly, only a small number of candidates began with the completely wrong hypotheses H$_0$: $\mu = 12.48$, H$_1$: $\mu > 12.48$. Those who used the critical region method needed to be careful with accuracy as the critical value and sample mean differ only in the fourth significant figure; in fact it is always wise in this type of test to calculate critical values to plenty of decimal places.

The fact that an apparently trifling change in the test produces the opposite conclusion is perhaps a commentary on the "significance level" approach to testing, and in the real world the use of p-values (which here would be 0.0488 and 0.0505) has become common.

Although many gave their conclusions in an admirably correct way, a statement that "the mean lifetime of animals _has been reduced"_ is wrong; candidates who wrote this were answering a different question.

Q.8(iii)   As so often, a question testing the use of the Central Limit Theorem revealed misunderstandings. As usual the question required *either* a necessary *or* a sufficient condition (here a necessary condition) and the mark scheme penalised the quotation of the wrong condition (though an answer such as "We need to use the CLT because the parent distribution is not stated to be normal, but we can use it as n is large" was accepted, the key word being "can").

There seems to be at least two widespread misunderstandings about the CLT. One is that a large sample makes the parent distribution normally distributed; put like this it is obviously wrong. Another is expressed by the answer "We do not need to use the Central Limit Theorem as it is a large sample" (or "a continuous distribution"); what do these candidates think that the CLT actually says? It may be worth emphasising that we are talking about two different variables (a single observation X, and the mean of n observations $\bar{X}$), and that these two variables have different distributions. The statement of the CLT is that it does not matter what the distribution of a single observation is; if the sample size is large enough, the distribution of the sample mean $\bar{X}$ is approximately normal.

Q.9   Many candidates made a good attempt at this question. As usual the key step in answering a question about Type I or Type II errors is to find the critical region (using the "correct" value of the parameter, here $\lambda = 11$). Many candidates did this more-or-less correctly, though many gave the CR as ≥ 19 rather than ≥ 20, and as noted above it was also common to obtain a critical value but not then to state the corresponding critical region correctly. Also, as in Question 5, it is necessary to quote any relevant probabilities in order to justify the stated CR. As usual, weaker candidates attempted to find a "critical region" from $\lambda = 14$, or simply calculated P($\geq 14 \mid \lambda = 11$). Some of those who obtained a CR as ≥ 20 (or ≥ 19) then gave their final answer as the probability of being in the critical region, rather than outside it. Nevertheless many completely correct answers were seen, and almost everyone knew that this was a Type II error. 0.923(5).

4734 Probability & Statistics 3

General Comments:

There were 282 candidates, slightly fewer than recent years. As usual, many produced extremely good scripts.
There was no evidence of candidates running out of time.
The modal mark for every part was full marks.

The standard of the stated hypotheses has improved, but there are still candidates who do not make it clear that the hypotheses are about the population from which the sample was drawn. This is best done by using standard symbols eg μ.

The standard of final conclusions was very good. There were few over-assertive conclusions.

The standard of presentation of many scripts was lamentable. Candidates should be aware that if their written answers are difficult to read, they will not be given any marks for them. Also, many candidates lost accuracy marks because they misread their own figures.

Comments on Individual Questions:

1 Almost all gained full marks, but some candidates read the tables incorrectly. A few candidates used the normal approximation to the Poisson distribution.

2 Over half the candidates gained full marks. The most common error was in the critical value.
A few candidates decided to combine classes, because one of the **observed** values was less than 5.

3(i) Most candidates scored full marks. Those who did not usually used t instead of the correct z-value.

3(ii) As 3(i).

4 (i) About half the candidates gained full marks.Those who did not often used an incorrect critical value. Some used the biased estimate of variance. A handful used a 2- sample test.

4(ii) About half the candidates knew the correct assumption.

5(i) Most candidates gained full marks. Those who did not usually stated that $a+b$=130.

5(ii) Full marks were obtained by about half the candidates. Use of σ^2 instead of the correct $2\sigma^2$
was the usual error. Others obtained the correct answer from incorrect working
eg $\dfrac{0-10}{2\sigma^2}=1.825$.

6(i) Again, roughly half the candidates gave a fully correct answer. Errors included poor hypotheses, using the pooled estimate of variance and using an incorrect critical value.

6(ii) About two-thirds of the candidates gained full marks. A common, major error was to omit '-0.04'.

7(i)   Almost all the candidates answered correctly.

7(ii)   As 7(i).

7(iii)   Roughly half the candidates gave a fully correct answer. Some did not use f(*x*) to obtain the expected values. A common error was to use eg 281, 611 so that *O-E* was always 2 or -2. Some gave inadequate hypotheses, others used an incorrect critical value.

8(i)   Once again, about half the candidates gave a fully correct answer. Some candidates used an incorrect formula for the surface area of a sphere, despite this formula being in the formula booklet. Marks were also lost for using the incorrect case for R/r and A/a.

   Some forfeited the first two method marks and immediately substituted $\sqrt{\dfrac{a}{4\pi}}$ into $\dfrac{r}{10}$.

8(ii)   Most candidates knew that they needed to substitute 200π into their CDF. Those who had the correct CDF usually obtained the correct answer.

4735 Probability & Statistics 4

General Comments:

There were 66 candidates, similar to recent years. As usual, many produced extremely good scripts. Approximately one-third scored 70 or more out of 72.
There was no evidence of candidates running out of time.
The modal mark for every part was full marks, except for Q5(iii).

Comments on Individual Questions:

Question No.

1(i)   About two-thirds of the candidates gained full marks. Those who did not usually lost marks for hypotheses which referred to samples or an over-assertive conclusion.

1(ii)   Almost all the candidates gave an acceptable response.

2   About three-quarters of the candidates gave a fully correct response. Those who did not usually lost marks for hypotheses which referred to samples or an over-assertive conclusion. Numerical errors were very rare.

3(i)   This question proved to be the easiest question on the paper. Only a handful of candidates made errors.

3(ii)   Most candidates answered correctly, but a sizeable minority incorrectly stated that Cov=0 $\Rightarrow$ *X* and *Y* are independent.

3(iii) This question was almost always answered correctly.

4(i)   About three-quarters of the candidates answered correctly. Some weaker candidates did not know what to do.

4(ii)   Most candidates knew what to do, but algebraic errors prevented many from gaining full marks. The identity $b^3 - a^3 = (b-a)(b^2 + ab + a^2)$ was not widely known.

5(i)   Four-fifths of the candidates obtained full marks.

5(ii)   Three-quarters of the candidates obtained full marks. Some candidates did not fully understand the concept of mutual exclusiveness.

5(iii) This was by far the most difficult question on the paper. Half the candidates gained no marks. Many did not realise that $P(A \cup B)=0.9 \Rightarrow P(A' \cap B' \cap C)=0.1$. A few found the value of *x* correctly, but could make no further progress. Candidates who drew Venn diagrams did better than those who wrote down many equations but did not know what to do with them.

6(i)   Almost always answered correctly. A few stated that P(1)=0.24 , not 0.48.

6(ii)   Almost all knew what to do, and most answered correctly.

6(iii) All the candidates attempted the question by differentiation, usually successfully. One candidate realised that *X* and *Y* were independent binomial distributions and used *np* and *npq* to check the answers.

6(iv) Almost always answered correctly.

7(i) Half scored full marks. CDF of $S = a^{10} s^{-10}$ was a common mistake.

7(ii) Candidates who answered part (i) correctly usually went on to gain full marks in this part also. Candidates who made the common mistake in part (i), lost further marks in this part.

7(iii) Three-fifths of candidates scored full marks. Most of the others had little idea what to do.

7(iv) Almost all gained this mark.

4736 Decision Mathematics 1

General Comments:

Most candidates were able to complete the paper in the time allowed, a few had wasted time by copying out tables or diagrams unnecessarily. Some candidates struggled to articulate answers to the questions asking for an explanation.

Candidates must read the questions carefully as several lost marks for not answering the question that had been asked. Working should always be shown as otherwise an incorrect answer cannot even earn method marks.

Some candidates used an additional booklet for 'rough working' even though they had been provided with extra answer space on page 12 of the answer booklet, centres should discourage them from doing this as it can lead to work being missed when it has not been labelled with the question number and part. The answer space is designed to be sufficient to accommodate the answer, extra space should only be needed when a candidate has had to start again.

Comments on Individual Questions:

Question 1
Mostly done well, a common mistake was to list the order in which the vertices were joined in rather than to list the arcs in the order that they were added. A few candidates used nearest neighbour instead of Prim's algorithm.

Question 2
Many correct answers. Most candidates realised that if bubble sort had been used to sort the list, into increasing order, then 53 would have been at the right-hand end of the list after the first pass instead of 26. Part (ii), in which candidates were asked to deduce the original list, was a variation on the usual question which most candidates coped with successfully. The first-fit decreasing packing in part (v) showed the usual problem with some candidates forgetting to always go back to the first bin that had sufficient capacity. Some candidates managed to misread their own list of numbers.

Question 3
Several candidates forgot the non-negativity constraints on x, y and z, and some had the inequality signs the wrong way round. A few candidates gave inequalities that also included the slack variables. The application of the Simplex algorithm was usually done well and most candidates were able to interpret the given final tableau.

Question 4
Candidates coped well with unpicking the issues with the various claims that the characters in this question made about their graphs. Quite a few thought that Molly had claimed that her graph was simply connected, when the actual problem was that the sum of the vertex orders in any graph cannot be odd. Most candidates recognised that the vertex of order 6 was causing the problem in Holly's graph and many were able to expand on exactly what the problem was. A few candidates claimed that the total vertex order was too great for Holly's graph to be simple, but the maximum case would have been the complete graph K_6 which has 15 arcs and therefore total vertex order 30.

The minimum caser for Olly's graph was a tree, giving a total vertex order of 10, the requirement that the orders are all even was not introduced until part (b). Finally, Polly's graph on 6 vertices needed to be simply connected and Eulerian (the same as Olly's) and also have 10 arcs, these requirements meant that the vertex orders needed to be, respectively, less than 6, greater than 0, even and have total 20. The only six values that fit this description are (in any order) two 2's and

four 4's. Most candidates were able to draw a connected graph with six vertices but some of the graphs did not fit all the requirements. A few candidates claimed that it was not possible to have 10 arcs and that the greatest possible number of arcs was 9, in fact the greatest number would have been 12 – but that was not the question that was being asked here.

Question 5
Most candidates carried out Dijkstra's algorithm correctly, although a few deleted the working values so that they could not be read. The output from Dijkstra's algorithm showed the shortest distance from *G* to each of the other vertices, in particular *G* to *K* = 7 and *G* to *L* = 8, the arc *KL* (or *LK*) added another 2 and then the shortest routes to *N* are from *L* directly or from *K* by travelling *K-L-N*. These two are the only possibilities, giving *G-H-K-L-N* = 7+2+4 = 13 and *G-J-I-L-K-L-N* = 8+2+6 =16, the first of these was the shortest route. In part (iii), the vast majority of candidates used the route inspection algorithm, but a few opted for 'trying out a route and hoping for the best'. The question specified that the route inspection algorithm was to be used and that working needed to be shown. Several candidates gave the routes that were repeated and not the individual arcs that needed to be repeated and many counted the number of arcs at *L* (including the repeated arcs) instead of halving to get the number of times that *L* was passed through. There was some speculation as to whether the name Percy Li was an anagram, but this was not intended to be the case.

Question 6
Candidates were asked to explain why the given constraint was needed, many realised that it was a scaled version of the time constraint $20x + 15y \le 100$, but either gave this inequality without reference to time or said that it came from the time available without showing how. The cost constraint of $99x + 104y \le 600$ was often achieved, although some candidates had clearly expected 'easier' numbers. The requirement that there was at least one of each type meant that *x* and *y* both needed to be $\ge$ 1. In part (iii), some candidates gave values for both *x* and *y* instead of a single total, a few candidates thought that the attendants were the guests, but this did not affect their ability to answer the question. Part (iv) required some careful reading to understand that what was being asked for was a solution that maximised the total number of attendants at minimum total cost, the solution for this part needed to be interpreted in the context of the wedding scenario.

Question 7
Generally done well, apart from poor arithmetic and a lack of tours starting and ending at *Y* in the last part. Some candidates queried whether the time to visit the start/end vertex needed to be included in the tour time and a few candidates decided to visit this attraction at the start and again at the end.

4737 Decision Mathematics 2

General Comments:

The candidates were able to complete the paper in the time allowed and there was a distinct improvement in the neatness of the written work from most candidates.

Candidates must read the questions carefully as several lost marks for not doing exactly what had been asked in the question. Working always needs to be shown as otherwise an incorrect answer cannot even gain method marks.

Some candidates used an additional booklet for 'rough working' even though they had been provided with extra answer space on page 12 of the answer booklet, centres should discourage them from doing this as it can lead to work being missed when it has not been labelled with the question number and part. The answer space is designed to be sufficient to accommodate the answer, extra space should only be needed when a candidate has had to start again.

Comments on Individual Questions:

Question 1
In parts (i) and (ii) candidates were asked to write down the shortest alternating path and hence write down the (incomplete or complete) matching. Those who did not give the alternating path had not done what was asked. Some candidates ignored the line for the alternating path above the space for the matching and instead chose to squeeze the alternating path in at the side of the matching. The explanations for part (iii) were generally good and usually led to a correct complete matching, although some candidates repeated the matching they had found in part (ii).

Question 2
Most candidates were able to interpret the capacities of the arcs *FT* and *DT* in part (i) and then use them, along with *ET*, to find the value of the cut. Some candidates seemed to think that the value of a cut is the sum of the actual flows across the cut rather than the maximum possible flow across the cut (without reference to restrictions elsewhere in the network).

The updating of the route in part (ii)(a) was generally done well and the answers were usually able to be read, even when candidates went on to do a second augmentation on the same diagram. Finding the second flow augmenting route, in part (ii)(b), tested some of the candidates who did not realise that the labelling procedure accommodates 'backflow' without needing to break the answer up into pieces. Some candidates thought that the flow given in part (ii)(a) must have been wrong, but this was because they did not appreciate how the labelling procedure works.

Question 3
Many correct answers to part (i), some candidates did not state what they were doing (in particular, those who subtracted a constant from the entire matrix before starting). The majority of the candidates correctly modified the matrix by adding a dummy column. The values in the dummy column should be equal and large, so that it is an undesirable choice. Using small dummy values resulted in an inefficient solution, as did reducing columns before rows. Only a few candidates converted the problem to a maximisation, these gained partial credit but got into a mess in part (ii).

The augmenting in part (ii) was usually correct, some candidates augmented in two stages by adding 3 to each crossed out row and each crossed out column and then subtracting 3 throughout as a separate stage, this was inefficient as the two operations can be done as a single stage. A small number of candidates augmented by 3 apart from the entries that were crossed out twice where they only augmented by 1, this is wrong. Some candidates did not say which technician was to be allocated to which task, even though this was specifically requested in the question. Candidates should be encouraged to show their working, for example showing the values being added as well as the total.

In part (iii) some candidates used the reduced cost matrix from part (ii), setting A=L and then looking at the position of the 0's, ignoring what happened to the dummy column. Most candidates realised that they had to start again and either removed column A and row L to give a 3×3 matrix and applied the Hungarian algorithm, or they stated that Dee would not be used and checked the cost of B=S, C=T against the cost of B=T, C=S. As in part (ii), some candidates did not say which technician was to be allocated to which task and some did not show enough working for it to be evident which values they had used from the table and that they had subtracted these from their cost from part (ii) reduced by 1. A few candidates said that Amir's cost should be reduced by £3 but did not answer the question of what his new cost should be.

Question 4
Candidates were asked to use an example in part (i) not just describe what a zero-sum game is. Some gave vague descriptions without saying what each player chose and how much they gained or lost and a few completely misunderstood what the two tables represented. Despite this, part (ii) was usually correct, a few candidates wrote the points lost rather than the (negative) points won.

In part (iii), some candidates used the wrong table or used rows instead of columns, and some did not simplify their answers. This then created problems in part (iv) where sometimes candidates ended up with probabilities that did not add up to 1, or even with negative probabilities. There were three unknowns at this point, being the probabilities p and q and the expected winnings, candidates who used their calculators to solve the simultaneous equations often assumed that the expected winnings were 0 without any justification. Equating the expressions for the expected winnings when Colin plays scissors and when he plays rock was the quickest method as this eliminated two of the unknowns immediately.

Question 5
There were several correct answers to part (i), but also a number of candidates who thought that the answer should be other pairs, in particular B and D or A and B. In part (ii) several candidates omitted D as an immediate predecessor for F. The forward and backward passes in part (iii) were done well, apart from a small minority of candidates who could not handle the dummy activities. Some candidates wrote out the two critical paths rather than listing the critical activities.

Most candidates were able to draw correct schedules in parts (iv) and (v), although some candidates shaded out the boxes instead of labelling them with the activities and a few had activities starting too early or too late. In part (vi), A and B are on parallel critical paths so reducing the duration of A has no effect unless B is also reduced (similarly, reducing B has no effect unless either A or D is reduced). To cut 10 minutes from the duration it was necessary to reduce two activities on each critical path by 5 minutes each, because of the interaction between A and B, the cheapest way to do this was to reduce H and J at a cost of £1000. Some candidates tried to reduce a single activity by 10 minutes contrary to the information given in the question and some reduced non-critical activities which had slack of at least 5 minutes (and hence no time was saved).

Question 6
The candidates who understood the relationship between the action value and the suboptimal values from the previous stage were usually able to score full marks in part (i). A few candidates forgot to write the maximum value or left (4; 0) out of their route. Candidates coped well with parts (ii) and (iii), apart from a few numerical slips. The network usually had the correct structure and vertex labels, but the arcs were not always shown as being directed. Some candidates thought the scenario in part (iv) was unrealistic, but finding a minimax of absolute differences is similar to minimising a standard deviation to find the most consistent outcome. Some candidates were inconsistent in the order of the pair of values being maximised in part (iv), which made following through errors from part (iii) difficult, and some did not show their working values and what the maximum was. A significant number of candidates added the pair of values rather than finding the maximum in each row of the working column and some candidates lost marks through silly arithmetic slips.

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