

WST02/01: Statistics S2

Question Number	Scheme	Marks
Q1	<p>(a) A population is collection of all items</p> <p>(b) (A random variable) that is a function of the sample which contains no unknown quantities/parameters.</p> <p>(c) The voters in the town Percentage/proportion voting for Dr Smith</p> <p>(d) Probability Distribution of those voting for Dr Smith from all possible samples (of size 100)</p>	<p>B1 (1)</p> <p>B1 (1)</p> <p>B1 (2)</p> <p>B1 (1)</p> <p><b>[5]</b></p>
	<p><b>Notes</b></p> <p>(a) <b>B1</b> – collection/group <b>all</b> items – need to have /imply all eg entire/complete/every</p> <p>(b) <b>B1</b> – needs <u>function/calculation(o.e.) of the sample/random variables/observations</u> <b>and no unknown quantities/parameters(o.e.)</b> NB do not allow unknown variables e.g. “A calculation based <u>solely</u> on observations from a given sample.” B1 “A calculation based <u>only</u> on known data from a sample” B1 “A calculation based on known observations from a sample” B0</p> <p>(c) <b>B1</b> – Voters  Do not allow 100 voters.  <b>B1</b> – percentage/ proportion voting (for Dr Smith) the <b>number</b> of people voting (for Dr Smith) Allow 35% of people voting (for Dr Smith) Allow 35 people voting (for Dr Smith) Do <b>not</b> allow 35% or 35 alone</p> <p>(d) <b>B1</b> – answers must include all three of these features (i) All possible samples, (ii) their associated probabilities, (iii) context of voting for Dr Smith.  e.g. “It is all possible values of the percentage and their associated probabilities.” B0 no context</p>	<p>Solely/only imply no unknown quantities</p>



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Q3	<p>Method 1</p> $P(X > 6) = \frac{1}{6}$ $P(X < 4) = \frac{1}{2}$ $\text{total} = \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$	<p>Method 2</p> $P(4 < X < 6) = \frac{1}{3}$ $1 - \frac{1}{3} = \frac{2}{3}$	<p>Method 3</p> $P(X > 6) = \frac{1}{6}$ $Y \sim U[3,9] \quad P(Y > 6) = \frac{1}{2}$ $\text{total} = \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$	<p>B1 M1</p> <p>A1</p> <p>M1dep B A1</p> <p>(5)</p> <p>[5]</p>
<p>Notes</p> <p><b>Methods 1 and 2</b></p> <p><b>B1</b> for 6 and 4 (allow if seen on a diagram on <math>x</math>-axis)</p> <p><b>M1</b> for <math>P(X &gt; 6)</math> or <math>P(6 &lt; X &lt; 7)</math>; or <math>P(X &lt; 4)</math> or <math>P(1 &lt; X &lt; 4)</math>; or <math>P(4 &lt; X &lt; 6)</math></p> <p>Allow <math>\leq</math> and <math>\geq</math> signs</p> <p><b>A1</b> <math>\frac{1}{6}</math>; or <math>\frac{1}{2}</math>; <math>\frac{1}{3}</math> must match the probability statement</p> <p><b>M1</b> for adding their “<math>P(X &gt; 6)</math>” and their “<math>P(X &lt; 4)</math>” or 1 - their “<math>P(4 &lt; X &lt; 6)</math>” dep on getting first B mark</p> <p><b>A1</b> cao <math>\frac{2}{3}</math></p> <p><b>Method 3 <math>Y \sim U[3, 9]</math></b></p> <p><b>B1</b> for 6 with <math>U[1,7]</math> and 6 with <math>U[3,9]</math></p> <p><b>M1</b> for <math>P(X &gt; 6)</math> or <math>P(6 &lt; X &lt; 7)</math> or <math>P(6 &lt; Y &lt; 9)</math></p> <p><b>A1</b> <math>\frac{1}{6}</math>; or <math>\frac{1}{2}</math>; must match the probability statement</p> <p><b>M1</b> for adding their “<math>P(X &gt; 6)</math>” and their “<math>P(Y &gt; 6)</math>” dep on getting first B mark</p> <p><b>A1</b> cao <math>\frac{2}{3}</math></p>				

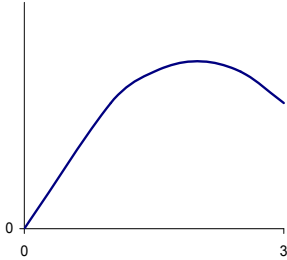
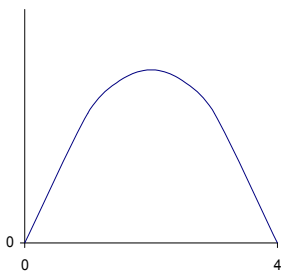
Question Number	Scheme	Marks
<p>Q4 (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	$\frac{4}{9}(m^2 + 2m - 3) = 0.5$ $m^2 + 2m - 4.125 = 0$ $m = \frac{-2 \pm \sqrt{4 + 16.5}}{2}$ $m = 1.26, -3.264$ <p>(median =) 1.26</p> <p>Differentiating <math>\frac{d\left(\frac{4}{9}(x^2 + 2x - 3)\right)}{dx} = \frac{4}{9}(2x + 2)</math></p> $f(x) = \begin{cases} \frac{8}{9}(x + 1) & 1 \leq x \leq 1.5 \\ 0 & \text{otherwise} \end{cases}$ <p><math>P(X \geq 1.2) = 1 - F(1.2)</math>  <math>= 1 - 0.3733</math>  <math>= \frac{47}{75}, 0.6267</math></p> <p><math>(0.6267)^4 = 0.154</math></p>	<p>M1</p> <p>M1</p> <p>A1 (3)</p> <p>M1 A1</p> <p>B1ft (3)</p> <p>M1</p> <p>A1 (2)</p> <p>M1 A1 (2)</p> <p>[10]</p>
<p><u>Notes</u></p>		
<p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p><b>M1</b> putting <math>F(x) = 0.5</math>  <b>M1</b> using correct quadratic formula. If use calc need to get 1.26 (384...)  <b>A1</b> cao 1.26 must reject the other root.                      If they use Trial and improvement they have to get the correct answer to gain the second M mark.</p> <p><b>M1</b> attempt to differentiate. At least one <math>x^n \rightarrow x^{n-1}</math>  <b>A1</b> correct differentiation  <b>B1</b> must have both parts- follow through their <math>F'(x)</math> Condone &lt;</p> <p><b>M1</b> finding/writing <math>1 - F(1.2)</math> may use/write <math>\int_{1.2}^{1.5} \frac{8}{9}(x + 1)dx</math> or <math>1 - \int_1^{1.2} \frac{8}{9}(x + 1)dx</math>                      or <math>\int_{1.2}^{1.5}</math> "their <math>f(x)</math>" <math>dx</math>. Condone missing <math>dx</math></p> <p><b>A1</b> awrt 0.627</p> <p><b>M1</b> (c)<sup>4</sup> If expressions are not given you need to check the calculation is correct to 2sf.  <b>A1</b> awrt 0.154 or 0.155</p>	

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<p><b>Q5</b></p> <p><b>(a)</b></p> <p><b>(b)</b></p> <p><b>(i)</b></p> <p><b>(ii)</b></p> <p><b>(c)</b></p>	<p>Connecting occurs at random/independently, singly or at a constant rate</p> <p>Po (8)</p> <p><math>P(X = 0) = 0.0003</math></p> <p><math>P(X \geq 4) = 1 - P(X \leq 3)</math>  <math>= 1 - 0.0424</math>  <math>= 0.9576</math></p> <p><math>H_0: \lambda = 4</math> (48) <math>H_1: \lambda &gt; 4</math> (48)</p> <p>N(48,48)</p> <p>Method 1</p> $P(X \geq 59.5) = P\left(Z \geq \frac{59.5 - 48}{\sqrt{48}}\right)$ $= P(Z \geq 1.66)$ $= 1 - 0.9515$ $= 0.0485$ <p>Method 2</p> $\frac{x - 0.5 - 48}{\sqrt{48}} = 1.6449$ <p><math>x = 59.9</math></p> <p><math>0.0485 &lt; 0.05</math></p> <p>Reject <math>H_0</math>. Significant. 60 lies in the Critical region</p> <p>The number of failed connections at the first attempt has increased.</p>	<p>B1 (1)</p> <p>B1</p> <p>M1A1</p> <p>M1</p> <p>A1 (5)</p> <p>B1</p> <p>M1 A1</p> <p>M1 M1 A1</p> <p>A1</p> <p>M1</p> <p>A1 ft (9)</p> <p>[15]</p>
<p><b>(a)</b></p> <p><b>(b)</b></p> <p><b>(i)</b></p> <p><b>(ii)</b></p> <p><b>(c)</b></p>	<p><b>Notes</b></p> <p><b>B1</b> Any one of randomly/independently/singly/constant rate. Must have context of connection/logging on/fail</p> <p><b>B1</b> Writing or using Po(8) in (i) or (ii)</p> <p><b>M1</b> for writing or finding <math>P(X = 0)</math>  <b>A1</b> awrt 0.0003</p> <p><b>M1</b> for writing or finding <math>1 - P(X \leq 3)</math>  <b>A1</b> awrt 0.958</p> <p><b>B1</b> both hypotheses correct. Must use <math>\lambda</math> or <math>\mu</math></p> <p><b>M1</b> identifying normal</p> <p><b>A1</b> using or seeing mean and variance of 48</p> <p>These first two marks may be given if the following are seen in the standardisation formula : 48 and <math>\sqrt{48}</math> or awrt 6.93</p> <p><b>M1</b> for attempting a continuity correction (Method 1: <math>60 \pm 0.5</math> / Method 2: <math>x \pm 0.5</math>)</p> <p><b>M1</b> for standardising using their mean and their standard deviation and using either Method 1 [<math>59.5, 60</math> or <math>60.5</math>. accept <math>\pm z</math>.] Method 2 [<math>(x \pm 0.5)</math> and equal to a <math>\pm z</math> value]</p> <p><b>A1</b> correct z value awrt <math>\pm 1.66</math> or <math>\pm \frac{59.5 - 48}{\sqrt{48}}</math>, or <math>\frac{x - 0.5 - 48}{\sqrt{48}} = 1.6449</math></p> <p><b>A1</b> awrt 3 sig fig in range 0.0484 – 0.0485, awrt 59.9</p> <p><b>M1</b> for “reject <math>H_0</math>” or “significant” maybe implied by “correct contextual comment”</p> <p>If one tail hypotheses given follow through “their prob” and <math>0.05, p &lt; 0.5</math></p> <p>If two tail hypotheses given follow through “their prob” with <math>0.025, p &lt; 0.5</math></p> <p>If one tail hypotheses given follow through “their prob” and <math>0.95, p &gt; 0.5</math></p> <p>If two tail hypotheses given follow through “their prob” with <math>0.975, p &gt; 0.5</math></p> <p>If no <math>H_1</math> given they get M0</p> <p><b>A1 ft</b> correct contextual statement followed through from their prob and <math>H_1</math>, need the words <u>number of failed connections/log ons</u> has <u>increased</u> o.e.</p> <p>Allow “there are more failed connections”</p> <p>NB A correct contextual statement <b>alone</b> followed through from their prob and <math>H_1</math> gets M1 A1</p>	

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Q6 (a)	2 outcomes/faulty or not faulty/success or fail A constant probability Independence Fixed number of trials (fixed $n$ )	B1 B1  (2)
(b)	$X \sim B(50, 0.25)$ $P(X \leq 6) = 0.0194$ $P(X \leq 7) = 0.0453$ $P(X \geq 18) = 0.0551$ $P(X \geq 19) = 0.0287$  CR $X \leq 6$ and $X \geq 19$	M1     A1 A1 (3)
(c)	$0.0194 + 0.0287 = 0.0481$	M1A1 (2)
(d)	8(It) is not in the Critical region or 8(It) is not significant or $0.0916 > 0.025$ ; There is evidence that the probability of a faulty bolt is 0.25 or the company's claim is correct.	M1; A1ft (2)
(e)	$H_0: p = 0.25$ $H_1: p < 0.25$ $P(X \leq 5) = 0.0070$ or CR $X \leq 5$ $0.007 < 0.01$ , 5 is in the critical region, reject $H_0$ , significant. There is evidence that the probability of faulty bolts has decreased	B1B1 M1A1  M1 A1ft (6) <b>[15]</b>
	Notes (a) <b>B1 B1</b> one mark for each of any of the four statements. Give first B1 if only one correct statement given. No context needed. (b) <b>M1</b> for writing or using $B(50, 0.25)$ also may be implied by both CR being correct. Condone use of P in critical region for the method mark. <b>A1</b> $(X) \leq 6$ o.e. [0,6] DO NOT accept $P(X \leq 6)$ <b>A1</b> $(X) \geq 19$ o.e. [19,50] DO NOT accept $P(X \geq 19)$ (c) <b>M1</b> Adding two probabilities for two tails. Both probabilities must be less than 0.5 <b>A1</b> awrt 0.0481 (d) <b>M1</b> one of the given statements followed through from their CR. <b>A1</b> contextual comment followed through from their CR. NB A correct contextual comment <b>alone</b> followed through from their CR will get M1 A1 (e) <b>B1</b> for $H_0$ must use $p$ or $\pi$ (pi) <b>B1</b> for $H_1$ must use $p$ or $\pi$ (pi) <b>M1</b> for finding or writing $P(X \leq 5)$ or attempting to find a critical region or a correct critical region <b>A1</b> awrt 0.007/CR $X \leq 5$ <b>M1</b> correct statement using their Probability and 0.01 if one tail test or a correct statement using their Probability and 0.005 if two tail test. The 0.01 or 0.005 needn't be explicitly seen but implied by correct statement compatible with their $H_1$ . If no $H_1$ given M0 <b>A1</b> correct contextual statement follow through from their prob and $H_1$ . Need faulty bolts and decreased. NB A correct contextual statement <b>alone</b> followed through from their prob and $H_1$ get M1 A1	



Question Number	Scheme	Marks
(c)		B1 B1 (2)
(d)	mode = 2	B1 (1)
<b>[15]</b>		
(a) (i)	<p>Notes</p> <p><b>M1</b> for putting <math>f(y) \geq 0</math> or <math>f(3) \geq 0</math> or <math>ky(a-y) \geq 0</math> or <math>3k(a-3) \geq 0</math> or <math>(a-y) \geq 0</math> or <math>(a-3) \geq 0</math> or state in words the probability can not be negative o.e.  <b>A1</b> need one of <math>ky(a-y) \geq 0</math> or <math>3k(a-3) \geq 0</math> or <math>(a-y) \geq 0</math> or <math>(a-3) \geq 0</math> <b>and</b> <math>a \geq 3</math></p> <p>(ii) <b>M1</b> attempting to integrate (at least one <math>y^n \rightarrow y^{n+1}</math>) (ignore limits)  <b>A1 Correct integration.</b> Limits not needed. And equals 1 not needed.  <b>M1</b> dependent on the previous M being awarded. Putting equal to 1 and have the correct limits. Limits do not need to be substituted.  <b>A1</b> cso</p> <p>(b) <b>M1</b> for attempting to find <math>\int yf(y) dy</math> (at least one <math>y^n \rightarrow y^{n+1}</math>) (ignore limits)  <b>A1</b> correct Integration  <b>M1</b> <math>\int yf(y) = 1.75</math> and limits 0,3 dependent on previous M being awarded  <b>M1</b> subst in for <math>k</math>. dependent on previous M being awarded  <b>A1</b> cso 4  <b>B1</b> cao 1/9</p> <p>(c) <b>B1</b> correct shape. No straight lines. No need for patios.  <b>B1</b> completely correct graph. Needs to go through origin and the curve ends at 3.  <u>Special case:</u> If draw full parabola from 0 to 4 get B1 B0 Allow full marks if the portion between <math>x = 3</math> and <math>x = 4</math> is dotted and the rest of the curve solid.</p> 	
(d)	<b>B1</b> cao 2	