June 2005 6664 Core Mathematics C2 Mark Scheme

Question number	Scheme	Marks	
1.	$\frac{dy}{dx} = 4x - 12$ $4x - 12 = 0 \qquad x = 3$	B1	
	4x - 12 = 0 $x = 3$	M1 A1ft	
	y = -18	A1	(4)
			4
	M1: Equate $\frac{dy}{dx}$ (not just y) to zero and proceed to $x = \dots$ Alft: Follow through only from a linear equation in x . Alternative: $y = 2x(x-6) \Rightarrow \text{Curve crosses } x\text{-axis at 0 and 6} \qquad \text{B1}$ (By symmetry) $x = 3 \qquad \qquad \text{M1 Alft}$ $y = -18 \qquad \qquad \text{A1}$ Alternative: $(x-3)^2 \qquad \text{B1 for } (x-3)^2 \text{ seen somewhere}$ $y = 2(x^2-6x) = 2\{(x-3)^2-9\} \qquad x = 3$ M1 for attempt to complete square and deduce $x = \dots$ Alft $[(x-a)^2 \Rightarrow x = a]$ $y = -18 \qquad \text{A1}$		

Question number	Scheme	Marks	
2.	(a) $x \log 5 = \log 8$, $x = \frac{\log 8}{\log 5}$, $= 1.29$	M1, A1, A1 ((3)
	(b) $\log_2 \frac{x+1}{x}$ (or $\log_2 7x$)	B1	
	$\frac{x+1}{x} = 7$ $x =,$ $\frac{1}{6}$ (Allow 0.167 or better)		(3) 6
	 (a) Answer only 1.29: Full marks. Answer only, which rounds to 1.29 (e.g. 1.292): M1 A1 A0 Answer only, which rounds to 1.3: M1 A0 A0 Trial and improvement: Award marks as for "answer only". (b) M1: Form (by legitimate log work) and solve an equation in x. Answer only: No marks unless verified (then full marks are available). 		

Question number	Scheme	Marks	
3.	(a) Attempt to evaluate f(-4) or f(4)	M1	
	$f(-4) = 2(-4)^3 + (-4)^2 - 25(-4) + 12$ (= 128 + 16 + 100 + 12) = 0,		
	so is a factor.	A1	(2)
	(b) $(x+4)(2x^2-7x+3)$	M1 A1	
	(2x-1)(x-3)	M1 A1	(4)
			6
	(b) First M requires $(2x^2 + ax + b)$, $a \ne 0$, $b \ne 0$.		
	Second M for the attempt to factorise the quadratic.		
	Alternative: $(x+4)(2x^2+ax+b) = 2x^3 + (8+a)x^2 + (4a+b)x + 4b = 0, \text{ then compare coefficients to find } \underbrace{values}_{a=-7, b=3} \text{ [A1]}$		
	Alternative: Factor theorem: Finding that $f\left(\frac{1}{2}\right) = 0$, $\therefore (2x-1)$ is a factor [M1, A1]		
	n.b. Finding that $f\left(\frac{1}{2}\right) = 0$, $\therefore (x - \frac{1}{2})$ is a factor scores M1, A0 ,unless the factor 2 subsequently appears.		
	Finding that $f(3) = 0$, $\therefore (x-3)$ is a factor [M1, A1]		

Question number	Scheme	Marks	
4.	(a) $1+12px$, $+\frac{12\times11}{2}(px)^2$	B1, B1	(2)
	(b) $12p(x) = -q(x)$ $66p^2(x^2) = 11q(x^2)$ (Equate terms, or coefficients) $\Rightarrow 66p^2 = -132p$ (Equate terms, or coefficients)	M1	
	$\Rightarrow 00p - 132p \qquad \text{(Eqn. in } p \text{ of } q \text{ only})$	M1	
	$p=-2, \qquad q=24$	A1, A1	(4) 6
	(a) Terms can be listed rather than added. First B1: Simplified form must be seen, but may be in (b). (b) First M: May still have $\binom{12}{2}$ or $^{12}C_2$ Second M: Not with $\binom{12}{2}$ or $^{12}C_2$. Dependent upon having p 's in each term. Zero solutions must be rejected for the final A mark.		

Question number	Scheme	Marks	
5.	(a) $(x+10=)$ 60 α 120 (M: $180 - \alpha$ or $\pi - \alpha$) x = 50 $x = 110$ (or 50.0 and 110.0) (M: Subtract 10) (b) $(2x=)$ 154.2 β Allow a.w.r.t. 154 or a.w.r.t. 2.69 (radians) 205.8 (M: $360 - \beta$ or $2\pi - \beta$) x = 77.1 $x = 102.9$ (M: Divide by 2)	B1 M1	(4) (4) (8)
	(a) First M: Must be subtracting from 180 before subtracting 10. (b) First M: Must be subtracting from 360 before dividing by 2, or dividing by 2 then subtracting from 180. In each part: Extra solutions outside 0 to 180 : Ignore. Extra solutions between 0 and 180 : A0. Alternative for (b): (double angle formula) $1-2\sin^2 x = -0.9$ $2\sin^2 x = 1.9$ $\sin x = \sqrt{0.95}$ M1 $x = 77.1$ $x = 180 - 77.1 = 102.9$ M1 A1		

Question number	Scheme	Ma	ırks
6.	(a) Missing y values: 1.6(00) 3.2(00) 3.394	B1 B1	(2)
	(b) $(A =) \frac{1}{2} \times 4$, $\{(0+0)+2(1.6+2.771+3.394+3.2)\}$	B1, M1	A1ft
	= 43.86 (or a more accurate value) (or 43.9, or 44)		A1 (4)
	(c) Volume = $A \times 2 \times 60$	M1	
	$= 5260 \text{ (m}^3)$ (or 5270, or 5280)	A1	(2)
			8
	(b) Answer only: No marks.		
	(c) Answer only: Allow. (The M mark in this part can be "implied").		

Question number	Scheme	Marks	
7.	(a) $\frac{\sin x}{8} = \frac{\sin 0.5}{7}$ or $\frac{8}{\sin x} = \frac{7}{\sin 0.5}$, $\sin x = \frac{8\sin 0.5}{7}$	M1 A1ft	
	$\sin x = 0.548$	A1	(3)
	(b) $x = 0.58$ (α) (This mark may be earned in (a)).	B1	
	$\pi - \alpha = 2.56$	M1 A1ft	(3) 6
	(a) M: Sine rule attempt (sides/angles possibly the "wrong way round"). Alft: follow through from sides/angles are the "wrong way round". Too many d.p. given: Maximum 1 mark penalty in the complete question. (Deduct on first occurrence).		

Question number	Scheme	Marks	
8.	(a) Centre $(5, 0)$ (or $x = 5, y = 0$)	B1 B1	(2)
	(b) $(x \pm a)^2 \pm b \pm 9 + (y \pm c)^2 = 0 \implies r^2 = \dots \text{ or } r = \dots$, Radius = 4	M1, A1	(2)
	(c) $(1, 0)$, $(9, 0)$ Allow just $x = 1$, $x = 9$	B1ft, B1ft	(2)
	(d) Gradient of $AT = -\frac{2}{7}$	B1	
	(d) Gradient of $AT = -\frac{2}{7}$ $y = -\frac{2}{7}(x-5)$	M1 A1ft	(3)
			9
	(a) (0, 5) scores B1 B0.		
	 (d) M1: Equation of straight line through centre, any gradient (except 0 or ∞) (The equation can be in any form). A1ft: Follow through from centre, but gradient must be -2/7. 		

Question number	Scheme	Marks	
9.	(a) $(S =) a + ar + + ar^{n-1}$ "S =" not required. Addition required.	B1	
	$(rS =) ar + ar^2 + + ar^n$ " $rS =$ " not required (M: Multiply by r)	M1	
	$S(1-r) = a(1-r^n)$ $S = \frac{a(1-r^n)}{1-r}$ (M: Subtract and factorise) (*)	M1 A1cso	(4)
	(b) $ar^{n-1} = 35000 \times 1.04^3 = 39400$ (M: Correct a and r, with $n = 3, 4$ or 5).	M1 A1	(2)
	(c) $n = 20$ (Seen or implied)	B1	
	$S_{20} = \frac{35000(1 - 1.04^{20})}{(1 - 1.04)}$	M1 A1ft	
	(M1: Needs <u>any</u> r value, $a = 35000$, $n = 19$, 20 or 21).		
	(A1ft: ft from $n = 19$ or $n = 21$, but r must be 1.04).		
	= 1 042 000	A1	(4)
	() D1 A 1		10
	 (a) B1: At least the 3 terms shown above, and no extra terms. A1: Requires a completely correct solution. Alternative for the 2 M marks: M1: Multiply numerator and denominator by 1 - r. M1: Multiply out numerator convincingly, and factorise. 		
	(b) M1 can also be scored by a "year by year" method. <u>Answer only:</u> 39 400 scores full marks, 39 370 scores M1 A0.		
	(c) M1 can also be scored by a "year by year" method, with terms added. In this case the B1 will be scored if the correct number of years is considered. Answer only: Special case: 1 042 000 scores 2 B marks, scored as 1, 0, 0, 1 (Other answers score no marks).		
	Failure to round correctly in (b) and (c): Penalise once only (first occurrence).		

Question number	Scheme	Marks
10.	(a) $\int (2x + 8x^{-2} - 5) dx = x^2 + \frac{8x^{-1}}{-1} - 5x$	M1 A1 A1
	$\left[x^{2} + \frac{8x^{-1}}{-1} - 5x\right]_{1}^{4} = (16 - 2 - 20) - (1 - 8 - 5) \tag{= 6}$	M1
	x = 1: $y = 5$ and $x = 4$: $y = 3.5$	B1
	Area of trapezium = $\frac{1}{2}(5+3.5)(4-1)$ (= 12.75)	M1
	Shaded area = $12.75 - 6 = 6.75$ (M: Subtract either way round)	M1 A1 (8)
	(b) $\frac{dy}{dx} = 2 - 16x^{-3}$	M1 A1
	(Increasing where) $\frac{dy}{dx} > 0$; For $x > 2$, $\frac{16}{x^3} < 2$, $\therefore \frac{dy}{dx} > 0$ (Allow \ge)	dM1; A1 (4)
		12
	(a) Integration: One term wrong M1 A1 A0; two terms wrong M1 A0 A0. Limits: M1 for substituting limits 4 and 1 into a changed function, and subtracting the right way round.	
	Alternative:	D1
	x = 1: $y = 5$ and $x = 4$: $y = 3.5Equation of line: y - 5 = -\frac{1}{2}(x - 1) y = \frac{11}{2} - \frac{1}{2}x, subsequently used in$	B1
	integration with limits.	3 rd M1
	$\left(\frac{11}{2} - \frac{1}{2}x\right) - \left(2x + \frac{8}{x^2} - 5\right)$ (M: Subtract either way round)	4 th M1
	$\int \left(\frac{21}{2} - \frac{5x}{2} - 8x^{-2}\right) dx = \frac{21x}{2} - \frac{5x^2}{4} - \frac{8x^{-1}}{-1}$	1 st M1 A1ft A1ft
	(Penalise integration mistakes, not algebra for the ft marks)	
	$\left[\frac{21x}{2} - \frac{5x^2}{4} - \frac{8x^{-1}}{-1}\right]_1^4 = (42 - 20 + 2) - \left(\frac{21}{2} - \frac{5}{4} + 8\right)$ (M: Right way round)	2 nd M1
	Shaded area $= 6.75$	A1
	(The follow through marks are for the subtracted version, and again deduct an accuracy mark for a wrong term: One wrong M1 A1 A0; two wrong M1 A0 A0.)	
	Alternative for the last 2 marks in (b): M1: Show that $x = 2$ is a minimum, using, e.g., 2^{nd} derivative. A1: Conclusion showing understanding of "increasing", with accurate working.	