Teacher Support Materials 2008

Maths GCE

Paper Reference MS/SS1A/W

Question 1

1 The table shows the times taken, y minutes, for a wood glue to dry at different air temperatures, x °C.

x	10	12	15	18	20	22	25	28	30
y	42.9	40.6	38.5	35.4	33.0	30.7	28.0	25.3	22.6

(a) Calculate the equation of the least squares regression line $y = a + bx$. *(4 marks)*

(b) Estimate the time taken for the glue to dry when the air temperature is 21 °C. *(2 marks)*

Student Response

Question number

Leave blank

1a. $y = a + bx$ sub data into calc.

$a = 53.067\ldots$

$b = -1.00336\ldots$

$\therefore\ y = a + bx$

$\Rightarrow y = 53.07 - 1.00x$

4

1b. Sub in $x = 21°$ $\therefore\ y = 53.07 - 1.00 \times 21$

$y = 31.996\ldots$ $y = 32$ mins

2

6

SP (SLM) T63766

Commentary

A typical minimalist yet fully-correct answer. In part (a), the values of *a* and *b* are stated, obviously using a calculator's regression, followed by the equation. Evidence is provided, in part (b), of the substitution of $x = 21$ into the equation together with an answer that is then rounded sensibly.

Mark scheme

Q	Solution	Marks	Total	Comments
1(a)	b (gradient) = –1.01 to –1(.00) (b (gradient) = –1.05 to –0.95)	B2 (B1)		AWFW (–1.00337)
	a (intercept) = 53(.0) to 53.2 (a (intercept) = 52(.0) to 54(.0))	B2 (B1)	4	AWFW (53.06736)
	OR			
	Attempt at $\sum x$, $\sum x^2$, $\sum y$ and $\sum xy$ **or** Attempt at S_{xx} and S_{xy}	(M1)		180, 3986, 297 and 5552.7 386 and –387.3
	Attempt at **correct** formula for b (gradient)	(m1)		
	b (gradient) = –1.01 to –1(.00)	(A1)		AWFW
	a (intercept) = 53(.0) to 53.2	(A1)		AWFW
	Accept a and b interchanged only if then identified correctly in part (b), but B2 in (b) does **not** necessarily imply 4 marks in (a)			
(b)	When $x = 21$,			
	$y = 31.7$ to 32.2 ($y = 29.9$ to 34.1)	B2 (B1)	2	AWFW (32.0) AWFW
	Evidence of use of 21 in c's equation	(M1)		
	Special Cases (if seen):			
	$y = \frac{33.0 + 30.7}{2} = 31.8$ to 31.9	(B1)		AWFW; or equivalent
	$y = 31.85$ without working	(B1)		
	Total		**6**	

Question 2

2 A basket in a stationery store contains a total of 400 marker and highlighter pens. Of the marker pens, some are permanent and the rest are non-permanent. The colours and types of pen are shown in the table.

	Colour			
Type	**Black**	**Blue**	**Red**	**Green**
Permanent marker	44	66	32	18
Non-permanent marker	36	53	21	10
Highlighter	0	41	37	42

A pen is selected at random from the basket. Calculate the probability that it is:

(a) a blue pen; *(1 mark)*

(b) a marker pen; *(2 marks)*

(c) a blue pen or a marker pen; *(2 marks)*

(d) a green pen, given that it is a highlighter pen. *(2 marks)*

Student response

Question number		Leave blank
2 a	$\frac{160}{400} = 0.4$ ✓	B1
b	$\frac{160 + 120}{400} = \frac{280}{400}$ ✓ $= 0.7$ ✓	2
	Assumes B & M independent!	
c	~~marker~~ $0.4 + 0.7 - (0.4 \times 0.7)$	M0
	$= 0.82$ ✗	A0
d	$P(G/H) = \frac{42}{120}$ ✓ $= 0.35$ ✓	2 (5)

Commentary

Parts (a) and (b) are answered correctly with a more than adequate amount of working. The common error in part (c) is the use of the addition law for non mutually exclusive but independent events; at least the third term prevents an answer greater than unity as was seen on some scripts! The candidate has however then identified correctly the necessary values for the required conditional probability in part (d).

Mark Scheme

Q	Solution	Marks	Total	Comments
2(a)	P(*Blue*) $= \frac{160}{400} = 0.4$ or $\frac{2}{5}$ or $\frac{160}{400}$	B1	1	CAO; or equivalent
	In (b) to (d), method marks are for single fractions, or equivalents, only			
(b)	P(*Marker*) $= \frac{280}{400}$	M1		$270 \le$ Numerator ≤ 290 and Numerator < Denominator ≤ 400
	$= 0.7$ or $\frac{7}{10}$ or $\frac{280}{400}$	A1	2	CAO; or equivalent
(c)	P(B or M) = P($B \cup M$) =			
	$\frac{160+280-119}{400} = \frac{280+41}{400} = \frac{321}{400}$	M1		$290 \le$ Numerator ≤ 321 and Numerator < Denominator ≤ 400
	$= 0.802$ to 0.803 or $\frac{321}{400}$	A1	2	AWFW/CAO (0.8025)
(d)	P(*Green* | *Highlighter*) = P($G \mid H$) $= \frac{42}{120}$	M1		Numerator = 42 and $110 \le$ Denominator ≤ 120
	$= 0.35$ or $\frac{7}{20}$ or $\frac{42}{120}$	A1	2	CAO; or equivalent
	Total		**7**	

Question 3

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The table shows, for each of a sample of 12 handmade decorative ceramic plaques, the length, x millimetres, and the width, y millimetres.

Plaque	x	y
A	232	109
B	235	112
C	236	114
D	234	118
E	230	117
F	230	113
G	246	121
H	240	125
I	244	128
J	241	122
K	246	126
L	245	123

(a) Calculate the value of the product moment correlation coefficient between x and y. *(3 marks)*

(b) Interpret your value in the context of this question. *(2 marks)*

(c) On **Figure 1**, complete the scatter diagram for these data. *(3 marks)*

(d) In fact, the 6 plaques A, B, ..., F are from a different source to the 6 plaques G, H, ..., L.

With reference to your scatter diagram, **but without further calculations**, estimate the value of the product moment correlation coefficient between x and y for **each** source of plaque. *(2 marks)*

Student Response

Question number

Leave blank

3a) P.M.C.C = $\dfrac{\sum x_1 y_1 - \dfrac{(\sum x_1)(\sum y_1)}{n}}{\sqrt{\left(\sum x_1^2 - \dfrac{(\sum x_1)^2}{n}\right)\left(\sum y_1^2 - \dfrac{(\sum y_1)^2}{n}\right)}}$

$\sum x^2 = 681575$
$\sum x = 2859$
$\sum y^2 = 170342$
$\sum y = 1428$
$\sum xy = 340555$

M1

$\dfrac{340555 - 340221}{\sqrt{(418.28)(410)}}$

M1

$\dfrac{334}{414.104} = \underline{\underline{0.80656067}}$

A1

b) A fairly strong positive correlation, the larger the length the larger the width

2

Scatter diagram

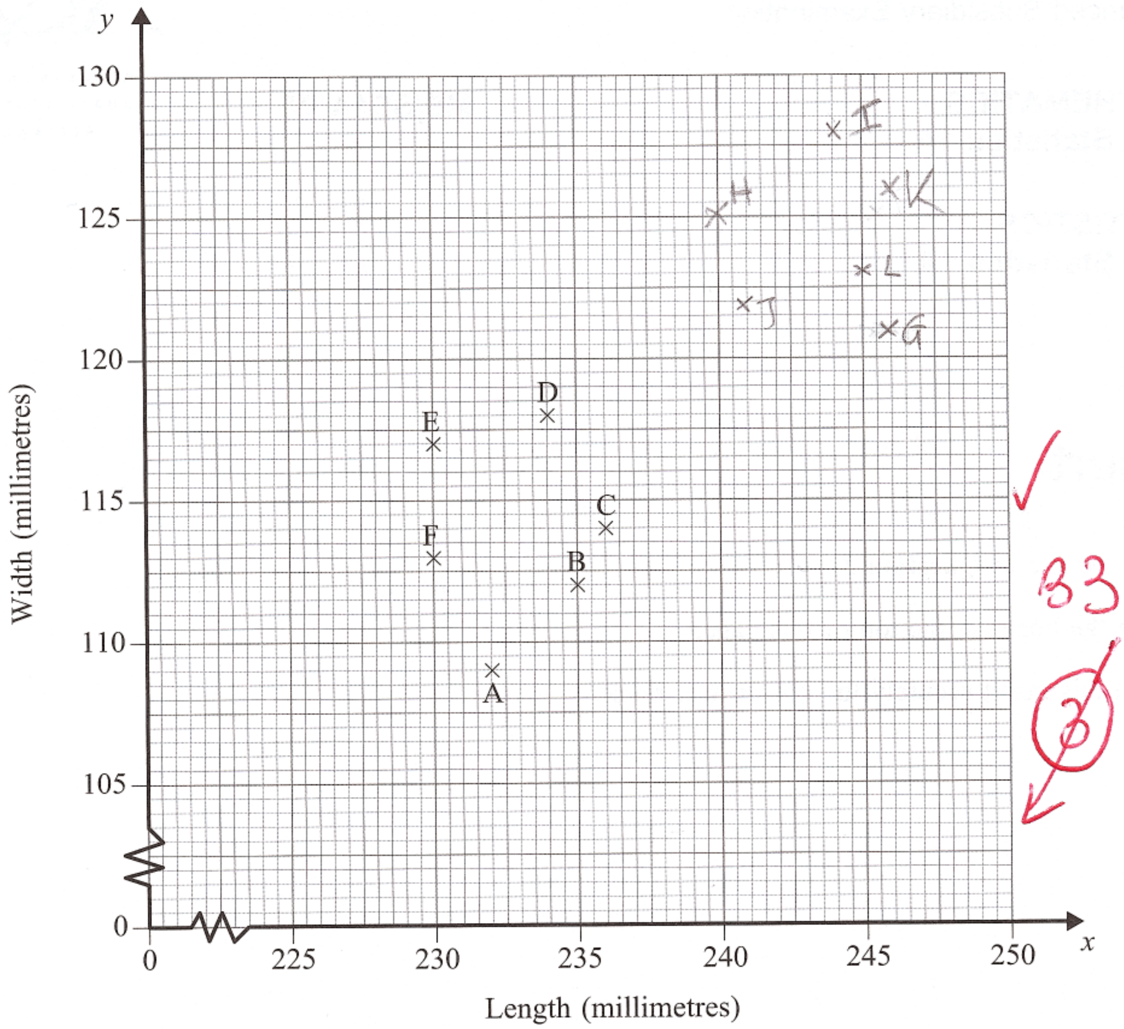
3

d) As there is noticably a positive correlation but not too strong as alot of the data is spread out i'll estimate the correlation to be around $\underline{\underline{0.5}}$ X

B0

B0

8



Commentary

In part (a), the candidate has obviously used a calculator to find the sums and sums of squares and then substituted these into a correct formula for *r* (as given in the Formulae Booklet) to obtain the full 3 marks. However, time would have been saved by simply writing down the answer direct from the calculator as did the vast majority of candidates. The answer to part (b) contains the necessary words of 'strong positive correlation' together with some reference to the question's context. The points are plotted and labelled correctly on the insert. As was the norm, the candidate has not realised that, for **each** source, the value of $r \approx 0$.

Mark Scheme

Q	Solution	Marks	Total	Comments
3(a)	r = 0.806 to 0.807	B3	3	AWFW (0.80656)
	(r = 0.8(0) to 0.81)	(B2)		AWFW
	(r = 0.7 to 0.9)	(B1)		AWFW
	OR			
	Attempt at $\sum x$, $\sum x^2$, $\sum y$, $\sum y^2$ and $\sum xy$			2859, 681575, 1428, 170342 and 340555
	or	(M1)		
	Attempt at S_{xx}, S_{yy} and S_{xy}			418.25, 410 and 334
	Attempt at **correct** formula for r	(m1)		
	r = 0.806 to 0.807	(A1)		AWFW
(b)	**Moderate/fairly strong/strong positive correlation** (relationship/association)	B1		Or equivalent; must qualify strength and indicate positive B0 for some/average/medium/very strong/etc
	between **length and width** of plaques	B1	2	Context; providing $0 < r < 1$
(c)	**Figure 1:**			
	6 correct labelled points	B3	3	
	(5 correct labelled points)	(B2)		Deduct 1 mark if not labelled
	(4 correct labelled points)	(B1)		
(d)	A to F: r = –0.2 to +0.2	B1		AWFW (–0.0275) No penalties for calculations
	Accept 'Zero' but not 'No' correlation			Statements must include a **single value** within range
	G to L: r = –0.2 to +0.2	B1	2	AWFW (–0.0196)
	Special Cases:			
	r = –0.2 to +0.2 with **no** sources	(B1)		AWFW
	r = –0.2 to +0.2 for **each/both** source(s)	(B2)		AWFW; or equivalent identification
	If B0 B0 but both values of r = –0.4 to +0.4	(B1)		AWFW
	Total		**10**	

Question 4

4 For the adult population of the UK, 35 per cent of men and 29 per cent of women do not wear glasses or contact lenses.

(a) Determine the probability that, in a random sample of 40 men, at most 15 do not wear glasses or contact lenses. *(3 marks)*

(b) Calculate the probability that, in a random sample of 10 women, exactly 3 do not wear glasses or contact lenses. *(3 marks)*

(c) (i) Calculate the mean and the variance for the number who **do** wear glasses or contact lenses in a random sample of 20 women. *(3 marks)*

(ii) The numbers wearing glasses or contact lenses in 10 groups, each of 20 women, had a mean of 16.5 and a variance of 2.50.

Comment on the claim that these 10 groups were **not** random samples. *(3 marks)*

Student Response

Question number

Leave blank

4: a) ~~chan~~ Probability of men not wearing contact lenses = 0.35

$B(40, 0.35)$

$P(X \leq 15)$

$= 0.6946$ ✓ 3

b) 0.24 of women ~~who~~ wear contact lenses

${}^{10}C_3 \times 0.29^3 \times 0.71^7$ ✓ ✓ ✓

$= 0.2661$ ✓ 3

C) i) $\mu = np$ $\mu = 20 \times 0.29$

$\mu = \underline{5.8}$ B0

Seen as can't have 0.8 of a women B0

5.8 rounded to 6

~~SD~~ Varience $= np(1-p)$

$= 20 \times 0.24(1-0.24)$

$5.8(0.71)$

$= 4.118$ ✓ B1

ii) This could not have been a random sample because the mean and varience are to different from the ~~actua~~ mean and varience of a known random sample. The mean was 10.7 higher than that of what i calculated from a known random sample, and the standard deviation was 1.618 lower. This ~~can~~ shows that the sample on which the claim was made was not a random sample.

Mean ✗ B0

Variance ✓ B1

Conclusion B0

8

Commentary

In common with most candidates, the answers to parts (a) & (b) are correct. However, in part (c), the candidate has apparently failed to read the question carefully, despite the embolden '**do**', and so used $p = 0.29$ rather than 0.71; here a costly (4-mark) error! As a result, marks are only available for the variance (same whether p is 0.29 or 0.71) and noting the discrepancy in the two variance/standard deviation values.

Mark Scheme

Q	Solution	Marks	Total	Comments
4	Binomial distribution	M1		Used somewhere in question
(a)	$M \sim \text{B}(40, 0.35)$	A1		Used; may be implied
	$\text{P}(M \le 15) = 0.69(0)$ to 0.696	A1	3	AWFW (0.6946)
(b)	$W \sim \text{B}(10, 0.29)$	B1		Used; may be implied
	$\text{P}(W = 3) = \binom{10}{3}(0.29)^3(0.71)^7$	M1		Stated; may be implied
	$= 0.266$ to 0.2665	A1	3	AWFW (0.2662) **Note**: $\text{B}(10, 0.3) \Rightarrow 0.2668$
(c)(i)	$n = 20 \quad p = 0.71$	B1		Stated or used; may be implied by 14.2
	Mean, $\mu = np = 14.2$	B1		CAO
	Variance, $\sigma^2 = np(1 - p)$ $= 4.11$ to 4.12	B1	3	AWFW (4.118)
(ii)	Mean of 16.5 is greater/different or $16.5/20 = 0.825$ is greater/different to 0.71	B1dep		Dependent on $\mu = 14.2$
	Means and variances are different	*(B2,1 dep)*		
	Variance of 2.50 is smaller/different	B1dep		Dependent on $\sigma^2 = 4.11$ to 4.12
	Suggests **claim** that groups are not random samples **is justified**	B1dep	3	Dependent on previous 2 marks Or equivalent
	Total		**12**	

Question 5

5 Vernon, a service engineer, is expected to carry out a boiler service in one hour.

One hour is subtracted from each of his actual times, and the resulting differences, x minutes, for a random sample of 100 boiler services have a mean, $\bar{x}$, of 1.90 and a standard deviation, s, of 3.32.

(a) Deduce, in minutes, the mean and the standard deviation of Vernon's actual service times for this sample. *(3 marks)*

(b) Construct a 98% confidence interval for the mean time taken by Vernon to carry out a boiler service. *(4 marks)*

(c) Vernon claims that, on average, a boiler service takes much longer than an hour.

Comment, with a justification, on this claim. *(1 mark)*

Student Response

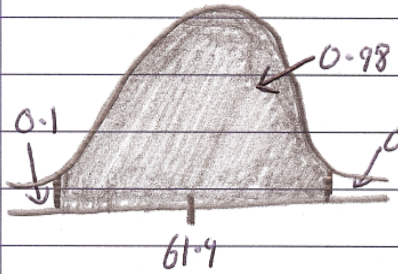
Question number | Leave blank

5a) ~~6t~~ 1·9 + 60 = 61·9 minutes mean actual service time

mean (μ) = 61·9 ✓ M1 A1

Standard deviation will stay the same at 3.32 mins ✓ B1

b) Standard error $= \dfrac{3\cdot32}{\sqrt{100}} = 0\cdot332$



S.e = 0·332

$z(0\cdot99) = 2\cdot3263$ ✓ B1

0·99

0·99 M1

$61\cdot9 \pm (2\cdot3263 \times 0\cdot332)$ A1

Times between 61·127 mins to 62·672 mins A1

c) Vernons claim that a service of a boiler takes longer than 1 hour ~~to correct~~ ~~item~~ seems to be correct, i am 98% certain that the average boiler time is between 61·127 and 62·672 mins and therefor 98% certain that the average time for a boiler ~~rep~~ service is greater than 1 hour. ✓ However the claim 'much longer' is ~~exa~~ exagerating a bit, they take slightly longer than 1 hour.

Contradictory. B0

7

Commentary

In part (a), the candidate has realised correctly that 60 is added to the mean but that no change is needed to the standard deviation. It was far too common to see 1 added to one or both values. The candidate's answer to part (b) is fully correct and it is good to see the use of a sketch in finding the correct z-value. In part (c), the answer contains a contradiction and so loses the 1 mark available.

Mark Scheme

Q	Solution	Marks	Total	Comments
5	$n = 100 \quad \bar{x} = 1.90 \quad s = 3.32$			
(a)	Mean $= 60 + \bar{x}$	M1		
	$= 61.9$	A1		CAO
	Standard deviation $= 3.32$	B1	3	CAO
(b)	$98\% \Rightarrow z = 2.32$ to 2.33 $(\Rightarrow t = 2.36$ to $2.37)$	B1		AWFW (2.3263) AWFW (2.364)
	CI for μ is $\bar{x} \pm z/t \times \frac{s}{\sqrt{n}}$	M1		Used; must have $\sqrt{n}$ with $n > 1$
	Thus $61.9 \pm 2.3263 \times \frac{3.32}{\sqrt{100}}$	A1$\sqrt{}$		$\sqrt{}$ on (a) and z/t only
	Hence $61.9 \pm (0.7$ to $0.8)$ or $(61.1$ to $61.2, 62.6$ to $62.7)$	A1	4	Accept $1.03 \pm (0.012$ to $0.013)$ AWFW Accept $(1.01$ to $1.02, 1.04$ to $1.05)$
(c)	$\bar{S}$ >> *1 hour or 60 minutes*: **Not valid** as UCL $\approx$ 1 hour (Accept Both limits $\approx$ 1 hour)	B1dep	1	Dependent on UCL $= 62.6$ to 62.7 or UCL $= 1.04$ to 1.05
	Total		**8**	

Question 6

6 The length, L centimetres, of *Slimline* bin liners may be modelled by a normal distribution with a mean of 69.5 and a standard deviation of 0.55 .

(a) Determine:

(i) $\mathrm{P}(L < 70)$; *(3 marks)*

(ii) $\mathrm{P}(69 < L < 70)$; *(3 marks)*

(iii) $\mathrm{P}(L = 70)$. *(1 mark)*

(b) Determine the maximum length exceeded by 90% of bin liners. *(4 marks)*

(c) The bin liners are sold in packets of 20, and those in each packet may be considered to be a random sample.

Determine the probability that:

(i) all the bin liners in a packet have lengths less than 70 cm; *(2 marks)*

(ii) the mean length of the bin liners in a packet is greater than 69.25 cm. *(4 marks)*

Student Response

Question number

Leave blank

6a) i $\dfrac{70-69.5}{0.55} = 0.909 \approx 0.91$

$\Phi\ 0.91 = 0.81859$ ✓ 3

ii) $\dfrac{69-69.5}{0.55} = -0.909 = (1-0.81859)$ ✓
$= 0.18141$

$P(69 < L < 70) = 0.81859 - 0.18141 = \underline{0.63718}$ ✓ ✓ ✓ 3

iii) $P(L=70) = 0.63718$ ✗ B0

b) $\dfrac{x-\mu}{0.55} = 1.2816$ sign B1 M1 A1

$x = 0.70488 + 69.5 = 70.20488$
$= 70.2$ ✗ A0

M1

ci) $0.81859^{20} = 0.01825 = 0.018$ ✓ ✓ A1

B0

ii) ~~$\frac{x-\mu}{0.55}$~~ ~~...~~ $n \times p = \text{mean}$ M0
No $\sigma/\sqrt{n}$ m0

$\dfrac{69.25}{20} = 3.4625$ probability A0
$= \underline{0.0049}$
or 0.0013

11

Commentary

The somewhat sketchy, but not uncommon, working to parts (a)(i) & (ii) did not hinder the candidate obtaining two correct answers. In part (a)(iii), the candidate did not realise that, as the normal distribution is continuous, the probability of any single value is zero; an error made by many candidates. In part (b), the again common error of an incorrect sign for the *z*-value lost 1 mark; 90% **exceeding** should have indicated that the resultant value must be **less than** the mean (69.5). The candidate's correct answer to part (c)(i), namely $\{\text{part(a)(i)}\}^{20}$, was rarely seen but, sadly, 0 out of 4 marks for part (c)(ii) was common for the reason as shown. When dealing with probabilities for a mean, the use of the correct standard error, $\left(\frac{\sigma}{\sqrt{n}}\right)$, is crucial to the remainder of the calculation.

Mark Scheme

Q	Solution	Marks	Total	Comments
6	Length $L \sim \mathrm{N}(69.5, 0.55^2)$			
(a)(i)	$\mathrm{P}(L < 70) = \mathrm{P}\left(Z < \frac{70 - 69.5}{0.55}\right) =$	M1		Standardising (69.5, 70 or 70.5) with 69.5 and ($\sqrt{0.55}$, 0.55 or 0.55^2) and/or ($69.5 - x$)
	$\mathrm{P}(Z < 0.91) =$	A1		0.91 AWRT; ignore sign
	0.818 to 0.82(0)	A1	3	AWFW (0.81835)
(ii)	$\mathrm{P}(69 < L < 70) =$ $\mathrm{P}(L < 70) - \mathrm{P}(L < 69) =$ $\mathrm{P}(Z < 0.91) - \mathrm{P}(Z < -0.91) =$	M1		Difference (70 − 69)
	$\mathrm{P}(Z < 0.91) - \{1 - \mathrm{P}(Z < 0.91)\} =$ $(0.81835) - (1 - 0.81835) =$	m1		Correct area change
	0.636 to 0.64(0)	A1	3	AWFW (0.63670)
(iii)	$\mathrm{P}(L = 70) = 0$	B1	1	CAO
(b)	0.90 (90%) $\Rightarrow z = -1.28$	B1		AWRT; ignore sign (−1.2816)
	$z = \frac{l - 69.5}{0.55}$	M1		Standardising l with 69.5 and 0.55; allow ($69.5 - l$)
	$= \pm 1.28(16)$	A1		Equating z-term to the z-value
	Hence l = 68.7 to 68.9	A1	4	AWFW; CSO (68.796)
(c)(i)	$\mathrm{P}(20L < 70) = \{(\mathrm{a})(\mathrm{i})\}^{20} =$	M1		Stated or used
	0.018 to 0.02(0)	A1	2	AWFW
(ii)	Variance of $\overline{L}_{20} = \frac{0.55^2}{20} = 0.0151(25)$ SD of $\overline{L}_{20} = \frac{0.55}{\sqrt{20}} = 0.123$	B1		CAO/AWRT; stated or used
	$\mathrm{P}\left(\overline{L}_{20} > 69.25\right) = \mathrm{P}\left(Z > \frac{69.25 - 69.5}{\sqrt{0.55^2/20}}\right)$	M1		Standardising 69.25 with 69.5 and 0.123; allow (69.5 − 69.25)
	$= \mathrm{P}(Z > -2.03) = \mathrm{P}(Z < 2.03) =$	m1		Correct area change
	0.978 to 0.98(0)	A1	4	AWFW (0.97896)
	Total		**17**	
	TOTAL		**60**	