Teacher Support Materials 2009

Maths GCE

Paper Reference MM1B

Question 1

1 Two particles, A and B, are moving on a smooth horizontal surface when they collide. During the collision, the two particles coalesce to form a single combined particle. Particle A has mass 3 kg and particle B has mass 7 kg.

Before the collision, the velocity of A is $\begin{bmatrix} 6 \\ -2 \end{bmatrix}$ m s^{-1} and the velocity of B is $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$ m s^{-1}.

(a) Find the velocity of the combined particle after the collision. *(3 marks)*

(b) Find the speed of the combined particle after the collision. *(2 marks)*

Student Response

$3\begin{bmatrix} 6 \\ -2 \end{bmatrix} + 7\begin{bmatrix} -1 \\ 4 \end{bmatrix} = 10v$ ✓ M1A1

$\begin{bmatrix} 18 \\ -6 \end{bmatrix} + \begin{bmatrix} -7 \\ 24 \end{bmatrix} = 10v$ 2

$\begin{bmatrix} 11 \\ 18 \end{bmatrix} = 10v$

$v = \begin{bmatrix} 1.1 \\ 1.8 \end{bmatrix}$

b) speed $= \sqrt{1.1^2 + 1.8^2} = 2.11$ m/s (3sf). ✓ M1A1F 2

4

Commentary

This candidate has written down a correct statement at the start of the answer, which secures the first two marks for the question. However in the second line of working, a simple arithmetic error, circled by the examiner causes the loss of the final accuracy mark for part (a).

The answer to part (b) is now clearly incorrect, but as the candidates has used the correct method and obtained the correct speed for the velocity found in part (a). The script shows how the examiner has awarded follow through marks in this case.

This example illustrates how candidates can lose marks through minor errors and how the follow through marks can be awarded. It also shows the importance of a clear statement or equation at the start of the question to ensure that partial marks are awarded.

Mark scheme

1(a)	$3\begin{bmatrix}6\\-2\end{bmatrix}+7\begin{bmatrix}-1\\4\end{bmatrix}=10\mathbf{v}$	M1		M1: Forming three term equation for conservation of momentum, but condone incorrect signs. Must see combined mass of 10.
		A1		A1: Correct equation with correct signs. Accept $3\begin{bmatrix}6\\-2\end{bmatrix}+7\begin{bmatrix}-1\\4\end{bmatrix}=3\mathbf{v}+7\mathbf{v}$ oe
	$\mathbf{v}=\frac{1}{10}\begin{bmatrix}11\\22\end{bmatrix}=\begin{bmatrix}1.1\\2.2\end{bmatrix}$	A1	3	A1: Correct velocity Consistent use of *mg* instead of *m* throughout deduct 1 mark
(b)	$v=\sqrt{1.1^2+2.2^2}$	M1		M1: Finding speed. Must be + inside square root.
	$v=2.46\ \text{ms}^{-1}$	A1F	2	A1F: Correct speed for their velocity Accept $1.1\sqrt{5}$ or $\frac{11\sqrt{5}}{10}$ or 2.45 or AWRT 2.46
	Total		**5**	

Question 2

2 A lift is travelling upwards and accelerating uniformly. During a 5 second period, it travels 16 metres and the speed of the lift increases from $u\,\mathrm{m\,s^{-1}}$ to $4.2\,\mathrm{m\,s^{-1}}$.

(a) Find u. *(3 marks)*

(b) Find the acceleration of the lift. *(3 marks)*

Student response

2a) $t = 5$

$s = 16$

$u = u$

$v = 4.2$

$v = u + at$.

$s = u^2 + \frac{1}{2}at$.

$v^2 = 2(u+v)t$

$17.64 = 2(u + 4.2)5$

$17.64 = 2(4.2u) \times 5$

$17.64 = 8.4u \times 5$

$17.65 = 42u$

$\frac{17.65}{42} = u$ $\quad u = 0.42\,\mathrm{ms^{-2}}$.

6

b) $v = u + at$

$4.2 = 0.42 + a \times 25$

$4.2 = 0.42 + 25a$

$4.2 - 0.42 = 25a$

$3.78 = 25a$

$\frac{3.78}{25} = a$. $\quad a = 0.15\,\mathrm{ms^{-2}}$

0

0

Commentary

This answer was produed by a candidate who has not been able to state the constant acceleration formulacorrectly. In part (a), the candidates states three formula, of which bunfortunatly only one is correct. One of the incorrect formulae is then used in part (a) to obtain an incorrect value for the initial velocity.

In part (b), the candidates quotes the required formula correctly, but then substitutes the time squared instead of the time.

This example shows how candidates who do not learn the required formula and who are not proficient in their application will lose many of the relatively straight forward marks that are available on the papers.

Mark Scheme

2(a)	$16=\frac{1}{2}(u+4.2)\times 5$ $32=5u+21$ $5u=11$ $u=\frac{11}{5}=2.2\ \text{ms}^{-1}$ OR First solution from (b) to find acceleration followed by any constant acceleration equation to find u: eg. $4.2=u+0.4\times 5$ $u=2.2$	M1A1 A1 (M1) (A1) (A1)	3	M1: Using a constant acceleration equation to find u with $v=4.2$ and $a\neq 9.8$. Could be derived from a velocity–time graph. A1: Correct equation A1: Correct value for u Eg $s=\frac{1}{2}(u+v)t$ followed by $16=(u+4.2)\times 5$ or similar scores M1A0
2(b)	$4.2=2.2+5a$ $5a=2$ $a=\frac{2}{5}=0.4\ \text{ms}^{-2}$ OR $16=2.2\times 5+\frac{1}{2}\times a\times 5^2$ $16=11+12.5a$ $a=\frac{5}{12.5}=0.4\ \text{ms}^{-2}$ OR $16=4.2\times 5-\frac{1}{2}\times a\times 5^2$ $16=21-12.5a$ $a=\frac{5}{12.5}=0.4\ \text{ms}^{-2}$ OR $4.2^2=2.2^2+2a\times 16$ $a=\frac{17.64-4.84}{32}=0.4\ \text{ms}^{-2}$	M1 A1F A1F (M1) (A1F) (A1F) (M1) (A1F) (A1F) (M1) (A1F) (A1F)	3	M1: Using a constant acceleration equation to find a with $u\neq 0$. A1F: Correct equation. Follow through for their incorrect u. A1F: Correct value for a, which must be >0. Follow through for their incorrect u. (If acceleration found correctly in part (a) and simply quoted as answer to (b) give full marks).
	Total		**6**	

Question 3

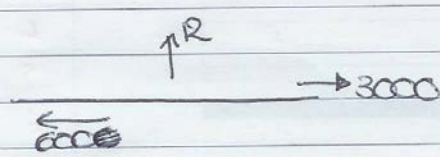
3 A car is travelling in a straight line on a horizontal road. A driving force, of magnitude 3000 N, acts in the direction of motion and a resistance force, of magnitude 600 N, opposes the motion of the car. Assume that no other horizontal forces act on the car.

(a) Find the magnitude of the resultant force on the car. *(2 marks)*

(b) The mass of the car is 1200 kg. Find the acceleration of the car. *(2 marks)*

Student Response

3



a) $R = 3000 - 600$
$= 2400\text{N}$.

2

b) $fr = ma$.
$3000 - (1200g + 600) = 1200a$. X
$3000 - 12360 = 1200a$.
$a = 7.8\text{m/s}$. X

0

2

Commentary

This candidate has calculated the resultant force correctly in part (a). The examiner has assumed that the R in the solution stands for the resultant force, not the normal reaction force which R appears to represent in the diagram.

In part (b) the candidate has included the weight in an equation which also involves the horizontal forces and so cannot find the acceleration.

This example illustrates someof the problems that the candidates have with drawing force diagrams and dealing with forces.

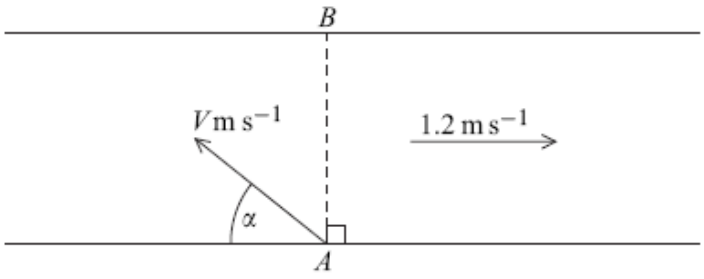
Mark Scheme

3(a)	Resultant Force = 3000 – 600 = 2400 N	M1 A1	2	M1: Difference between the two forces. A1: Correct magnitude of resultant force. Must be a positive answer. (600 – 3000 = –2400 scores M1A0)
(b)	$2400 = 1200a$ $a = \frac{2400}{1200} = 2\ \text{ms}^{-2}$	M1 A1	2	M1: Use of Newton's second Law to find acceleration. A1: Correct acceleration ($a = \frac{-2400}{1200} = -2\ \text{ms}^{-2}$ scores M1A0)
	Total		**4**	

Question 4

4 A river has parallel banks which are 16 metres apart. The water in the river flows at $1.2\,\mathrm{m\,s^{-1}}$ parallel to the banks. A boat sets off from one bank at the point A and travels perpendicular to the bank so that it reaches the point B, which is directly opposite the point A. It takes the boat 10 seconds to cross the river.

The velocity of the boat relative to the water has magnitude $V\,\mathrm{m\,s^{-1}}$ and is at an angle α to the bank, as shown in the diagram.



(a) Show that the magnitude of the resultant velocity of the boat is $1.6\,\mathrm{m\,s^{-1}}$. *(1 mark)*

(b) Find V. *(3 marks)*

(c) Find α. *(2 marks)*

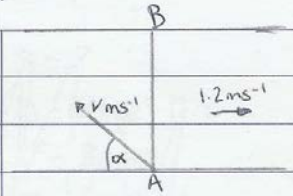
(d) State one modelling assumption that you needed to make about the boat. *(1 mark)*

Student Response

Question number

Leave blank

4)

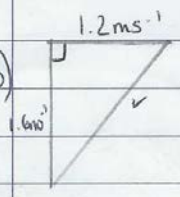


$AB = 16m$

$t = 10$ seconds

a) $V = \frac{16}{10}$ $\therefore$ $V = 1.6ms^{-1}$ ✓ 1

b)



$V = \sqrt{1.6^2 + 1.2^2}$

$V = \sqrt{4}$

$V = 2ms^{-1}$ ✓ 3

c) $\frac{2}{\sin 90} = \frac{1.2}{\sin \alpha}$

$2 = \frac{1.2}{\sin \alpha}$ $\therefore$ $\frac{1.2}{2} = \sin \alpha$

$0.6 = \sin \alpha$ M1 1

$\alpha = \sin^{-1} 0.6$

$\alpha = 36.9^\circ$ (3SF) 0

d) There is no water resistance other than the ~~flow~~ flow of the river. ✗ 5

Commentary

This candidate produces correct solutions to part (a) and (b) of the question and is clearly helped by the clear diagrams that were drawn to support these parts of the solution. The required division is shown clrealy in part (a).

In part (c) the candidate finds an angle correctly, but does nt realise that this is not the angle that the question has asked them to find. It is interesting that the candidate did not mark any angles, other than the right angle, on the diagram. It is quite possible that the candidate would have gained full marks if the angle that was found had been marked on the diagram.

The answer that this candidate gives in part (d) is not acceptable because the times and distances are specified in the question and these would take account of any resistance forces already.

This examples shows the importance of clear diagrams in candidates solutions.

Mark Scheme

4(a)	$v=\frac{16}{10}=1.6\ \text{ms}^{-1}$ AG	B1	1	B1: Printed result obtained from correct division. Must see 16 divided by 10.
(b)	$V^2=1.6^2+1.2^2$ $V=\sqrt{4}=2\ \text{ms}^{-1}$	M1A1 A1	3	M1: Equation to find V based on Pythagoras. Must involve addition of the squares of two components. A1: Correct equation A1: Correct V
4(c)	$\sin\alpha=\frac{1.6}{2}$ or $\frac{1.2}{2}$ $\alpha=53.1°$ OR $\cos\alpha=\frac{1.2}{2}$ or $\frac{1.6}{2}$ $\alpha=53.1°$ OR $\tan\alpha=\frac{1.6}{1.2}$ or $\frac{1.2}{1.6}$ $\alpha=53.1°$	M1 A1F (M1) (A1F) (M1) (A1F)	2	M1: Trigonometric equation to find α. A1F: Correct α. Follow through incorrect answer to (b). Ignore diagrams
(d)	The boat is a particle	B1	1	B1: Statement of particle assumption. Ignore any other assumptions.
	Total		**7**	

Question 5

5 A block, of mass 14 kg, is held at rest on a rough horizontal surface. The coefficient of friction between the block and the surface is 0.25. A light inextensible string, which passes over a fixed smooth peg, is attached to the block. The other end of the string is attached to a particle, of mass 6 kg, which is hanging at rest.

The block is released and begins to accelerate.

(a) Find the magnitude of the friction force acting on the block. *(3 marks)*

(b) By forming two equations of motion, one for the block and one for the particle, show that the magnitude of the acceleration of the block and the particle is $1.225\ \mathrm{m\,s^{-2}}$. *(5 marks)*

(c) Find the tension in the string. *(2 marks)*

(d) When the block is released, it is 0.8 metres from the peg. Find the speed of the block when it hits the peg. *(3 marks)*

(e) When the block reaches the peg, the string breaks and the particle falls a further 0.5 metres to the ground. Find the speed of the particle when it hits the ground. *(3 marks)*

Student Response

Leave blank

5.a. $\mu = 0.25$ $F = \mu R$

$R = 14g$

$F = 0.25 \times 14g$

$F = 0.25 \times 137.2$

$F = 34.3\,N$

3

b. $6g - T = 6a$

$+ \quad T - 34.3 = 14a$

$6g - 34.3 = 20a$

$58.8 - 34.3 = 20a$

$24.5 = 20a$

$a = 1.225\,ms^{-2}$

5

c. $F = ma$

$T - 34.3 = 14 \times 1.225$

$T = 14 \times 1.225 + 34.3$

$T = 17.15 + 34.3$

$T = 51.45\,N$

2

d. ~~$s = vt$~~

$v^2 = u^2 + 2as$ $v = ?$

$v^2 = 0 + 2 \times 1.225 \times 0.8$ $s = 0.8m$, $u = 0$, $a = 1.225$

$v^2 = 1.96$

$v = \sqrt{1.96}$

$v = 1.4\,ms^{-1}$

3

e. ~~$s = vt + \frac{1}{2}at^2$~~ $v = ?$

$v^2 = u^2 + 2as$ $s = 0.5m$, $u = 1.4\,ms^{-1}$, $a = 1.225$

$v = \sqrt{1.4^2 + 2 \times 1.225 \times 0.5}$

$v = \sqrt{3.185}$

$v = 1.785\,ms^{-1}$

0

13

Commentary

This candidate has produced a good solution. The working is clearly shown and all steps taken are indicated. This is particularly important in part (b), which requires the candidates to show that the acceleration has the given value. The candidates begins part (b) with clear equations of motions for each bodyand then solves them to obtain the required acceleration.

In part (e) the candidate uses the correct initial velocity, but makes the mistake of using the wrong acceleration.

This example illustrates a really good solution to a “Show that” type of question.

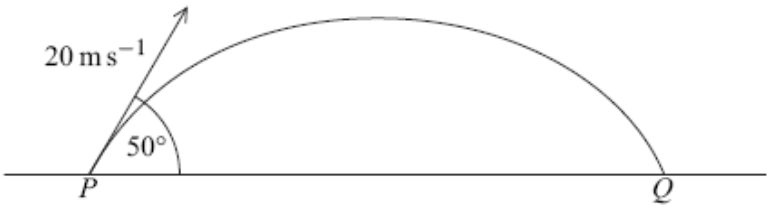
Mark Scheme

5(a)	$R=14\times9.8=(137.2)$	M1		M1: Finding the normal reaction. Accept $14g$.
	$F=0.25\times137.2$ OR $F=0.25\times14\times9.8$	M1		M1: Use of $F=\mu R$
	$F=34.3$ N	A1	3	A1: Correct friction Use of $g=9.81$ gives $R=137.3$ and $F=34.3$ so in this case do not penalise use of $g=9.81$.
(b)	$6g-T=6a$	M1A1		M1: Equation of motion for the particle, containing T, $6g$ or 58.8 and $6a$. A1: Correct equation with correct signs.
	$T-34.3=14a$	M1A1		M1: Equation of motion for the block, containing T, 34.3 or their F and $14a$. A1: Correct equation with correct signs.
	$6g-34.3=20a$			A1: Correct acceleration from correct working.
	$a=\dfrac{6g-34.3}{20}=1.225\ \text{ms}^{-2}$ AG	A1	5	If –1.225 is obtained from consistent working award 4 marks and if changed to +1.225 with an explanation, award full marks. Special Case: Whole string method $6g-34.3=20a$ OE $a=1.225$ award M1A1A1 Use of $g=9.81$ gives $a=1.228$ penalise use of $g=9.81$ by deducting 1 mark, but don’t penalise again on the same script.

5(c)	$T-34.3=14\times1.225$ $T=17.15+34.3=51.5$ N OR $6g-T=6\times1.225$ $T=6\times9.8-6\times1.225=51.5$	M1 A1 (M1) (A1)	2	M1: Use of either of candidates equations of motion to find tension, with $a=\pm1.225$ and their F (Method 1). A1: Correct tension Accept 51.45 or 51.4. Don't penalise use of g = 9.81 if already done in part (b).
(d)	$v^2=0^2+2\times1.225\times0.8$ $v=\sqrt{1.96}=1.4$ ms^{-1} OR $0.8=\frac{1}{2}\times1.225t^2$ $t=(1.1428)$ $v=1.225\times1.1428$ $=1.40$	M1A1 A1 (M1) (A1) (A1)	3	M1: Use of constant acceleration equation to find speed with $u=0$. A1: Correct equation A1: Correct speed AWRT 1.4 In method 2, no marks awarded for just finding t.
(e)	$v^2=1.4^2+2\times9.8\times0.5$ $v=3.43$ ms^{-1} OR $0.5=1.4t+4.9t^2$ $t=0.2071$ $v=1.4+9.8\times0.2071$ $=3.43$ ms^{-1}	M1 A1F A1F (M1) (A1F) (A1F)	3	M1: Use of constant acceleration equation to find speed with u = 1.4 or their answer to part (d), a = ±9.8 and s = 0.5. A1F: Correct equation. Follow through their answer to part (d). A1F: Correct speed. Don't penalise use of g = 9.81 if already done earlier in question. In method 2, no marks awarded for just finding t.
	Total		**16**	

Question 6

6 A ball is kicked from the point P on a horizontal surface. It leaves the surface with a velocity of $20\,\mathrm{m\,s^{-1}}$ at an angle of $50°$ above the horizontal and hits the surface for the first time at the point Q. Assume that the ball is a particle that moves only under the influence of gravity.



(a) Show that the time that it takes the ball to travel from P to Q is 3.13 s, correct to three significant figures. *(4 marks)*

(b) Find the distance between the points P and Q. *(2 marks)*

(c) If a heavier ball were projected from P with the same velocity, how would the distance between P and Q, calculated using the same modelling assumptions, compare with your answer to part (b)? Give a reason for your answer. *(2 marks)*

(d) Find the maximum height of the ball above the horizontal surface. *(3 marks)*

(e) State the magnitude and direction of the velocity of the ball as it hits the surface. *(2 marks)*

Student Response

Leave blank

6. a)

	s	u	v	a	t
→		20 cos 50	20 cos 50	0	
↑		20 sin 50	0	−9.8	

$V = U + at$

~~$0 = 20\sin 40 - 9.8t$~~

~~$20\sin 40 = 9.8t$~~ ~~$12.8558 = 9.8t$ $t =$~~

~~$t = 1.31$ seconds (for w only to maximum point)~~

~~Total time: 1.31 (×2) =~~

$V = U + at$

$V = 20\sin 50 - 9.8t$

$20\sin 50 = 9.8t$ ✓ M1A1dM1 3

$15.32 = 9.8t$ $t = 1.56$ seconds (only to maximum point)

total time = $1.56 \times 2 = 3.13$ seconds

b) $s = ut + \frac{1}{2}at^2$ 2

$s = 20\cos 50 \times 3.13$ ✓

$s = 40.2$ m

c) It wouldn't differ or affect the results in any way. Mass doesn't affect initial velocity or acceleration and therefore the distance between P and Q would remain the same. ✓ 2

d) ↑ $s = ut + \frac{1}{2}at^2$ 3

$s = 20\sin 50 \times 1.56 + \frac{1}{2} \times -9.8 \times 1.56^2$ ✓

$s = 23.9 - 11.92 = \cancel{\#}\ 12.0$ m

e)

	s	u	v	a	t	
↑		20sin50		+9.8	3.13	$v = u + at$
~~downwards~~						$v = 20\sin 50 + (9.8 \times 3.13)$
						~~20sin50 =~~ $v = 15.35$ ✓

stion nber

Leave blank

downwards. $\sqrt{15.35^2 + (20\cos 50)^2}$

$\sqrt{235.6225 + 165.27}$ ✗

$\sqrt{188.89} = 13.7$ m/s

0

(10)

Commentary

This candidate understands how to solve part (a) of the question, but fails to gain the final mark in this part because they include the statement 1.56 x 2 = 3.13. In order to have gained the final mark, the candidate should have produced a statement like 1.563 x 2 = 3.126 and then given the final answer as 3.13. In general, when asked to show a particular value to three significant figures, the candidates should give an answer to more than three significant figures and then round this to the required value.

The candidates gives good answers to parts (b), (c) and (d), although the examiner would have preffered to “the acceleration due to gravity” rather than just “acceleration” in part (c).

In part (e) The candidates had simply been expected to state the velocity with which the ball hits the surface. Many candidates however, went through the fairly long process of calculating the horizontal and vertical components of the velocity of the ball when its hits the surface and then using these to obtain the speed and direction of the ball at the time. The response from this candidate illustrates some interesting errors. In the first line of working the candidates writes “+” instead of “-“, as indicated by the examiner. However he does obtain the correct answer for this component, but subtracting rather than adding. In the line that begins with the word “downwards” the candidate shows what could be a correct expression for the speed if brackets were inseted as marked by the examiner. The line below is then correct, but unfortunately the candidate makes an arithmetic error.

This script again illustrates the need for candidates to fully justify their answers in “Show that” type questions and how candidates can be given the benefit of the doubt when they make slips in their working.

Mark Scheme

6(a)	$20\sin 50°t - 4.9t^2 = 0$ $t = \frac{20\sin 50°}{4.9}$ or $3.126... = 3.13$ s AG OR $0 = 20\sin 50° - 9.8t$ $t = \frac{20\sin 50°}{9.8} = 1.563$ $T = 2\times 1.563$ $= 3.13$	M1A1 dM1 A1 (M2) (A2)	4	M1: Equation to find time, with $y = 0$, $u = 20\sin 50°$ or $u = 20\cos 50°$ and ± 9.8 or $\pm g$. A1: Correct equation dM1: Solving for t. A1: Correct time from correct working. Must see division by 4.9 oe or more than 3sf Verification methods can only gain first 2 marks Special case $t = \frac{15.3}{4.9} = 3.12$ or 3.13 scores M1A1dM1A0 M2: doubling time to max height (could use cos instead of sin) but must use ± 9.8 or $\pm g$. A2: Correct time from correct working. Don't penalise use of $g = 9.81$ if already done earlier on script. Would obtain time as 3.12 seconds. Note: If using a memorised formula either 4 marks if final answer correct, 3 marks if substituted correctly, otherwise zero. Special case $T = 2\times 1.56 = 3.12$ or 3.13 scores M2A1
(b)	$PQ = 20\cos 50° \times 3.127 = 40.2$ m	M1 A1	2	M1: Calculation of range, could use sin instead of cos. A1: Correct range Accept 40.1
(c)	No change because a greater mass would not change the acceleration. OR Mass is not used in the equations.	B1 B1	2	B1: No change B1: Explanation following a correct statement.
6(d)	$0 = (20\sin 50°)^2 + 2\times(-9.8)s$ $s = \frac{(20\sin 50°)^2}{2\times 9.8} = 12.0$ m OR $t = \frac{3.13}{2} = 1.565$ $h = 20\sin 50° \times 1.565 - 4.9\times 1.565^2$ $= 12.0$	M1 A1 A1 (M1) (A1) (A1)	3	M1: Equation to find height, with $u = 20\sin 50°$ or $u = 20\cos 50°$ and ± 9.8 or $\pm g$ (and t between 1.56 and 1.57 if method 2 used). A1: Correct equation A1: Correct height. Accept 12 or 11.9 or AWRT 12.0 In method 2, no marks awarded for just finding t. Don't penalise use of $g = 9.81$ if already done earlier on script. Should still get 12. Note: If using a memorised formula either 3 marks if final answer correct, 2 marks if substituted correctly, otherwise zero.
(e)	20 ms^{-1} at 50° below the horizontal.	B1 B1	2	B1: Speed AWRT 20 B1: Direction AWRT 50°. Must indicate below, or down. Could be implied by a diagram.
	Total		**13**	

Question 7

7 A particle moves on a smooth horizontal plane. It is initially at the point A, with position vector $(9\mathbf{i} + 7\mathbf{j})$ m, and has velocity $(-2\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. The particle moves with a constant acceleration of $(0.25\mathbf{i} + 0.3\mathbf{j})\,\mathrm{m\,s^{-2}}$ for 20 seconds until it reaches the point B. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Find the velocity of the particle at the point B. *(3 marks)*

(b) Find the velocity of the particle when it is travelling due north. *(4 marks)*

(c) Find the position vector of the point B. *(3 marks)*

(d) Find the average velocity of the particle as it moves from A to B. *(2 marks)*

Student Response

7a $u = (-2\underline{i} + 2\underline{j})\,ms^{-1}$

$a = (0.25\underline{i} + 0.3\underline{j})\,ms^{-2}$

$t = 20$

$V = ?$

$V = u + at$

$V = (-2\underline{i} + 2\underline{j}) + 20(0.25\underline{i} + 0.3\underline{j})$

$V = (-2\underline{i} + 2\underline{j}) + 5\underline{i} + 6\underline{j}$

$V = (3\underline{i} + 8\underline{j})\,ms^{-1}$

3

7b Travelling due North when $\underline{i}$ component $= 0$

$0.25t\underline{i} = 3\underline{i}$

$0.25t = 3$

$t = \frac{3}{0.25} = 12$ secs

$V = u + at$

$V = (-2\underline{i} + 2\underline{j}) + 12(0.25\underline{i} + 0.3\underline{j})$

$V = -2\underline{i} + 2\underline{j} + 3\underline{i} + 3.6\underline{j}$

$V = (\underline{i} + 5.6\underline{j})\,ms^{-1}$

0

7c $R = ut + \frac{1}{2}at^2$

$u = (-2\underline{i} + 2\underline{j})\,ms^{-1}$

$a = (0.25\underline{i} + 0.3\underline{j})\,ms^{-2}$

$t = 20$

$= 20(-2\underline{i} + 2\underline{j}) + \frac{1}{2} \times 20^2 \times (0.25\underline{i} + 0.3\underline{j})$

$= -40\underline{i} + 40\underline{j} + 50\underline{i} + 60\underline{j}$

$= 10\underline{i} + 100\underline{j}$

Position Vector at B = Initial Vector + $10\underline{i} + 100\underline{j}$

$= (19\underline{i} + 107\underline{j})\,m$

3

Question number

Leave blank

7d A $(9\underline{i} + 7\underline{j})$

B $(19\underline{i} + 107\underline{j})$

distance AB $= B - A = 19\underline{i} + 107\underline{j} - 9\underline{i} + 7\underline{j}$

$= 10\underline{i} + 100\underline{j}$

Average Velocity $= \dfrac{\text{total distance}}{\text{time taken}} = \dfrac{10\underline{i} + 100\underline{j}}{20}$

Average Velocity $= (0.5\underline{i} + 5\underline{j})\ ms^{-1}$

2

8

Commentary

This candidate has produced good solutions to all parts of this question except for part (b).

Part (b) was found difficult by many candidates. This candidates, as did many others, begins with a correct statement about the **i** component being zero, but is unable to form an equation from which to beginto work. In this case the candidate has used 3, which is the **i** component of the velocity at time 20, but has not included the initial velocity. It is interesting to note that when the candidate calculates the velocity the answer includes a non-zero **i** component, but this does not seem to worry the candidate.

There are two other points worth mentioning about this sample. The candidate has clearly crossed out any working that he does not want to be marked. This avoids the averagingof marks rule which is applied when candidates supply two complete, but different solutions.

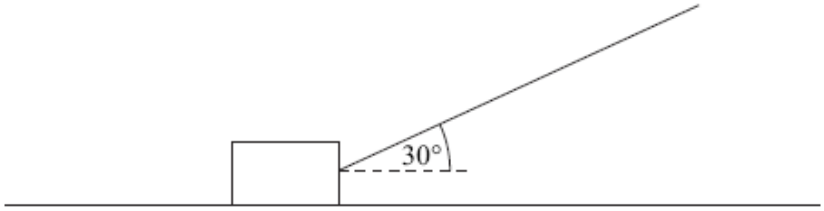
The other point can be seen in part (d). In the line that begins “distance AB” the candiadate should have put brackets around the initial position, but as the working is correct on the next line the candidate is not penalised. Clearly it is better for candidates to ensure that the working they present is always correct, but examiners will give candidates the benefit of the doubt in cases like this.

Mark Scheme

7(a)	$\mathbf{v}=(-2\mathbf{i}+2\mathbf{j})+(0.25\mathbf{i}+0.3\mathbf{j})\times 20$ $\mathbf{v}=3\mathbf{i}+8\mathbf{j}$	M1 A1 A1	 3	M1: Finding velocity using $\mathbf{v}=\mathbf{u}+\mathbf{a}t$. A1: Correct expression. A1: Correct velocity in simplest form.
(b)	$-2+0.25t=0$ $t=8$ s $\mathbf{v}=(2+0.3\times 8)\mathbf{j}=4.4\mathbf{j}$	M1A1 A1 A1	 4	M1: One component equal to zero (either **i** or **j** component). A1: Correct equation A1: Correct time A1: Correct velocity
(c)	$\mathbf{r}=(-2\mathbf{i}+2\mathbf{j})\times 20+\frac{1}{2}(0.25\mathbf{i}+0.3\mathbf{j})\times 20^2+(9\mathbf{i}+7\mathbf{j})$ **OR** $\mathbf{r}=\frac{1}{2}\left((-2\mathbf{i}+2\mathbf{j})+(3\mathbf{i}+8\mathbf{j})\right)\times 20+(9\mathbf{i}+7\mathbf{j})$ $\mathbf{r}=19\mathbf{i}+107\mathbf{j}$	M1 A1 A1	 3	M1: Finding position vector using a constant acceleration equation with or without the initial position with $t=20$. A1: Correct expression for position vector including initial position. A1: Correct position vector in simplest form.
(d)	$\mathbf{v}_{AVERAGE}=\frac{(19\mathbf{i}+107\mathbf{j})-(9\mathbf{i}+7\mathbf{j})}{20}$ $=\frac{10\mathbf{i}+100\mathbf{j}}{20}$ $=0.5\mathbf{i}+5\mathbf{j}$	M1 A1F	 2	M1: Finding average velocity based on change of position. Subtraction of initial position must be seen or implied. Division by 8 scores M0 A1F: Correct average velocity. Follow through incorrect answers from part (c). Allow $\frac{\mathbf{u}+\mathbf{v}}{2}$
	Total		**12**	

Question 8

8 The diagram shows a block, of mass 20 kg, being pulled along a rough horizontal surface by a rope inclined at an angle of 30° to the horizontal.



The coefficient of friction between the block and the surface is μ. Model the block as a particle which slides on the surface.

(a) If the tension in the rope is 60 newtons, the block moves at a constant speed.

(i) Show that the magnitude of the normal reaction force acting on the block is 166 N. *(3 marks)*

(ii) Find μ. *(4 marks)*

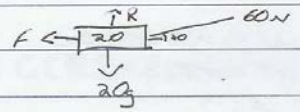
(b) If the rope remains at the same angle and the block accelerates at $0.8\ \text{m s}^{-2}$, find the tension in the rope. *(5 marks)*

Student Response

number

Leave blank

8.a.



i. $R = 20g - 60\sin 30$

$R = 196 - 30$

$R = 166N$

3

ii. $F = \mu R$ $\quad F = 60\cos 30$

$F = \mu R$

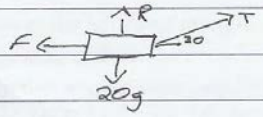
$60\cos 30 = 166\mu$

$51.96 = 166\mu$

$\mu = 0.313$

4

b.



$R = 20g - T\sin 30$

$F = T\cos 30 = 0.313(20g - T\sin 30)$ B1

1

$F = Ma$

$T - 0.313(20g - T\sin 30) = 20 \times 0.8$

$T - (61.348 - 0.313T\sin 30) = 16$

$T = 16 + (61.348 - 0.313T\sin 30)$

$T - T\cos 30 = 20 \times 0.8$

$T - T\cos 30 = 16$

$T = 16 + T\cos 30$ $\quad T = 13.9N$

$T = 16 + 0.866T$

$\frac{T}{0.866T} = 16$

$1.155T = 16$

8

Commentary

This was a very typical response to this question, in that the candidate gains full marks for part (a) of the question. Many candidates then scored no marks on part (b). This candidate is a little different, in that they do gain one mark for giving correct expressions for the normal reaction and friction force, but do not gain any more marks because they do not apply Newton's Second Law correctly.

This script illustrates how challenging part (b) of this question was for many candidates.

Mark Scheme

8(a)(i)	$20\times9.8=R+60\sin30°$ $(R=)20\times9.8-60\sin30°=166$ N AG	M1 A1 A1	3	M1: Equation or expression for normal reaction with mg or $20g$ or 196 and 60sin30° or 60cos30°. A1: Correct equation or expression with correct signs. A1: Correct value from correct working. Must be positive. Don't penalise use of $g = 9.81$ if already done earlier on script. Should still get 166, but from 166.2.
(ii)	$166\mu=60\cos30°$ $\mu=\dfrac{60\cos30°}{166}$ $=0.313$	M1 M1A1 A1	4	M1: Use of $F=\mu R$, with $R = 166$ or 166.2. Do not allow inequalities here. M1: Resolving horizontally with cos30° or sin30° oe. A1: Correct equation Examples: $166\mu=60$ M1M0A0 $166\mu=-60\cos30°$ M1M1A0 A1: Correct coefficient of friction.
(b)	$20\times0.8=T\cos30°-0.313(20\times9.8-T\sin30°)$ $T=\dfrac{20\times0.8+0.313\times20\times9.8}{\cos30°+0.313\sin30°}=75.6$ N	B1 M1 A1F dM1 A1F	5	B1: $20g-T\sin30°$ oe seen. M1: Three term equation of motion, where normal reaction is dependent on T. A1F: Correct equation dM1: Solving for T including factorisation. A1F: Correct tension. AWRT 75.6 Follow through incorrect values of μ from part (a). Don't penalise use of $g = 9.81$ if already done earlier on script. Should get 75.7. Allow 75.8 if intermediate values rounded.
	Total		**12**	