Teacher Support Materials 2008

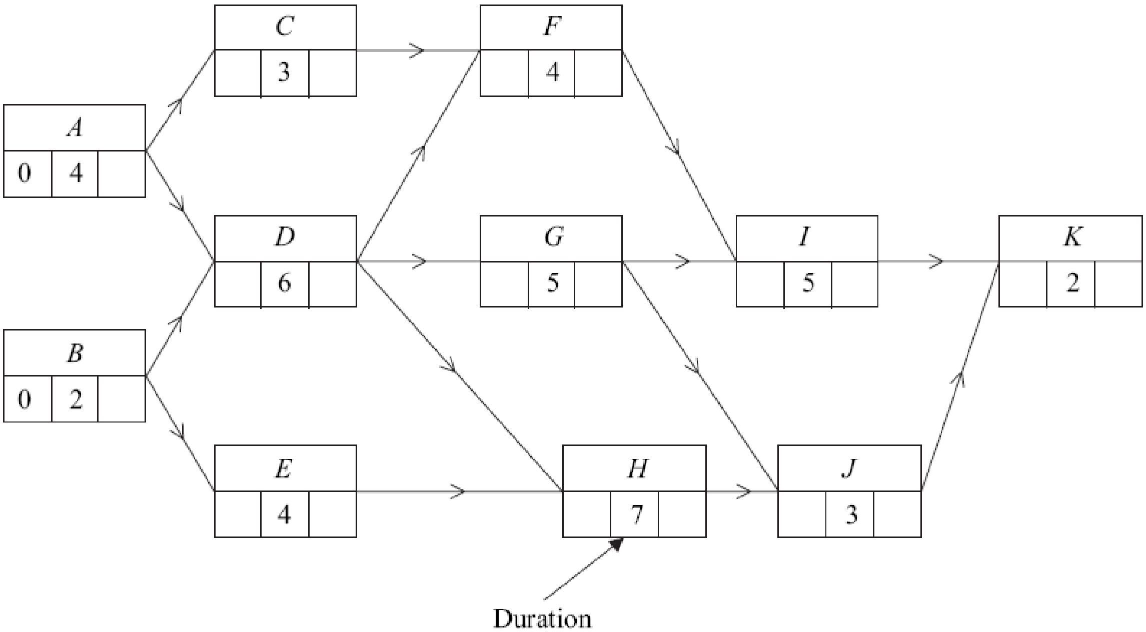
Maths GCE

Paper Reference MD02

Question 1

1 [**Figures 1 and 2**, printed on the insert, are provided for use in this question.]

The following diagram shows an activity network for a project. The time needed for each activity is given in days.



(a) Find the earliest start time and the latest finish time for each activity and insert their values on **Figure 1**. *(4 marks)*

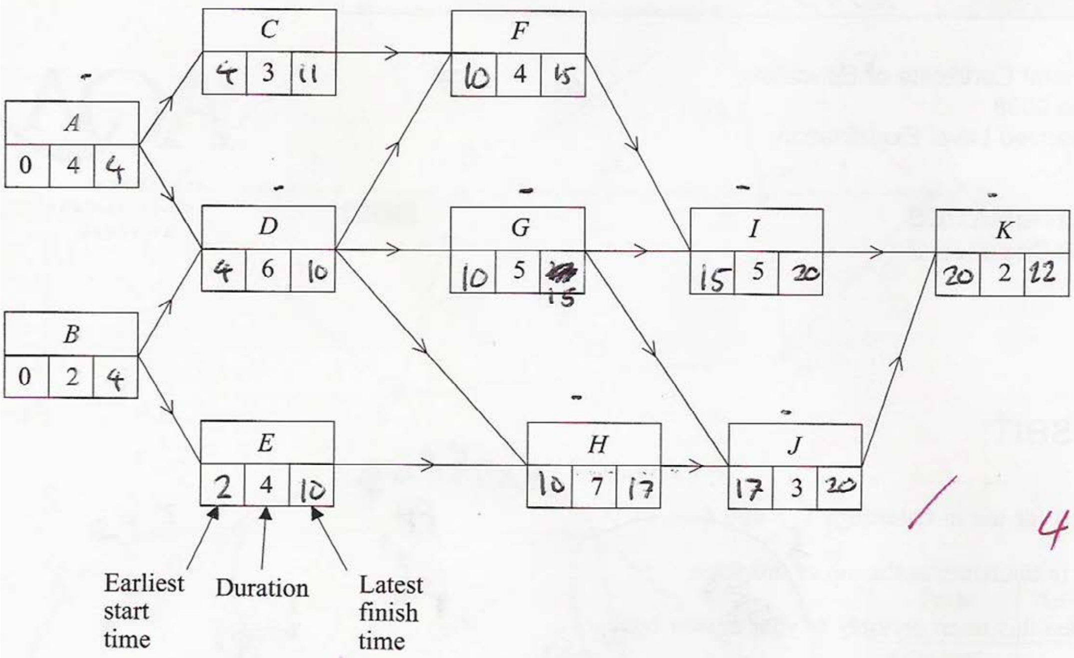
(b) Find the critical paths and state the minimum time for completion. *(3 marks)*

(c) On **Figure 2**, draw a cascade diagram (Gantt chart) for the project, assuming each activity starts as early as possible. *(3 marks)*

(d) Activity C takes 5 days longer than first expected. Determine the effect on the earliest start time for other activities and the minimum completion time for the project. *(2 marks)*

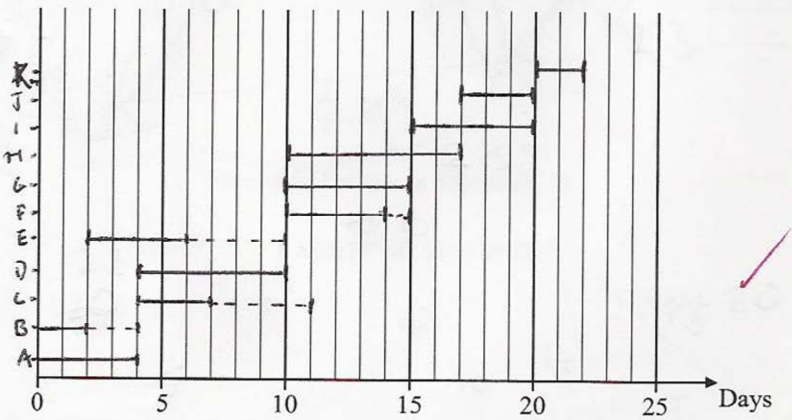
Student Response

Figure 1 (for use in Question 1)



4

Figure 2 (for use in Question 1)



3

7

Question number | Leave blank

1(b) Critical Paths = A D G I K and ~~ADHK~~ A D H J K 7

min completion time = 22 days 3

(d) C float time = 4 ∴ C will over run by 1 day

causing F, I, K to start 1 day later E0

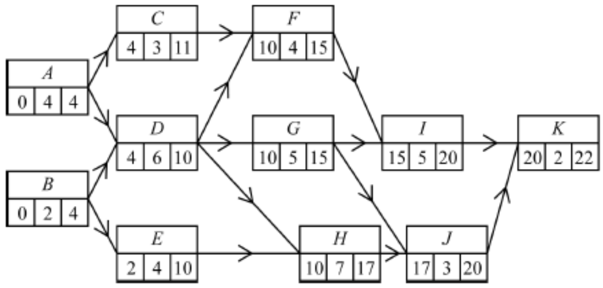
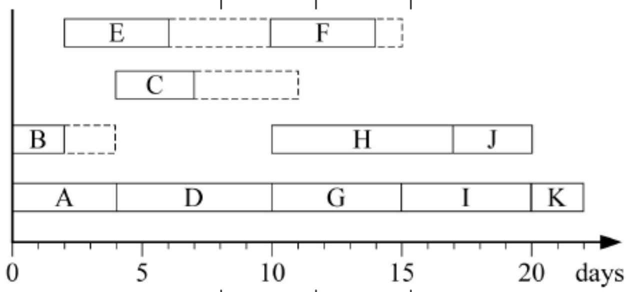
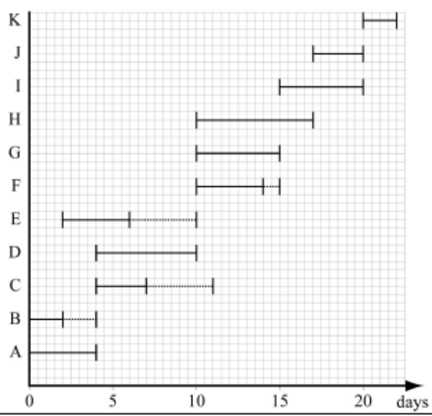
giving the min completion time of 23 B1 1

11

Commentary

(a) Full marks are scored for calculating the correct earliest start time and latest finish time for each event. The values are inserted in the correct places in Figure 1. The latest finish time for G was initially written as 17 but is clearly corrected to 15.
(b) The two critical paths are identified and the minimum completion time stated as 22 days.
(c) This candidate chooses to draw the cascade diagram by listing the events from A to K on the vertical axis and the float for each of the events B, C, E and F is indicated by a broken line. Other candidates chose to use horizontal blocks as in the mark scheme. Either type of diagram scores full marks.
(d) The candidate fails to explain that F is delayed by 2 days and cannot start until day 12 at the earliest. Despite this error the minimum completion time is correctly given as 23 days.

Mark scheme

Q	Solution	Marks	Total	Comments
1				
(a)	Earliest start times	M1		up to 2 errors ft
	All correct	A1		
	Latest finish times	M1		up to 2 errors ft from *K*
	Correct	A1	4	
(b)	Critical paths *A D G I K*	B1		
	A D H J K	B1		withhold if extra path such as *A D G J K* given
	Minimum time for completion = 22 days	B1	3	Must be stated – not just a value in box on insert
(c)				
	A, D, G, H, I, J, K correct	B1		
	B, C, E, F	B1		2 correct with slack /float or 4 correct & no slack
		B1	3	all correct with slack/float; withhold if slack not shown dotted etc
(d)	*F* starts after 12 days at earliest or *F* starts 2 days later *I* is now unable to start until after 16 days or *I* starts 1 day later	E1		
	Minimum completion time now 23 days – one extra day etc	B1	2	
	Total		**12**	

Question 2

2 The following table shows the scores of five people, Alice, Baji, Cath, Dip and Ede, after playing five different computer games.

	Alice	**Baji**	**Cath**	**Dip**	**Ede**
Game 1	17	16	19	17	20
Game 2	20	13	15	16	18
Game 3	16	17	15	18	13
Game 4	13	14	18	15	17
Game 5	15	16	20	16	15

Each of the five games is to be assigned to one of the five people so that the total score is maximised. No person can be assigned to more than one game.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $20 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table above from 20, and hence show that, by reducing **columns first** and then rows, the resulting table of values is as below.

3	1	1	1	0
0	4	5	2	2
4	0	5	0	7
5	1	0	1	1
5	1	0	2	5

(3 marks)

(c) Show that the zeros in the table in part (b) can be covered with one horizontal and three vertical lines. Hence use the Hungarian algorithm to reduce the table to a form where five lines are needed to cover the zeros. *(3 marks)*

(d) Hence find the possible allocations of games to the five people so that the total score is maximised. *(4 marks)*

(e) State the value of the maximum total score. *(1 mark)*

Student response

2). c) By taking the maximum value, then subtracting each number from this will allow the table to be maximised, and therefore the Hungarian Algorithm can then be used to allocate a game to a person, which will maximise the score. ? EO

b)

	A	B	C	D	E
1	3	4	1	3	0
2	0	7	5	4	2
3	4	3	5	2	7
4	7	6	2	5	3
5	5	4	0	4	5
		3		2	

	A	B	C	D	E
1	3	1	1	1	0
2	0	4	5	2	2
3	4	0	5	0	7
4	7	3	2	3	3
5	5	1	0	2	5

2

∴

	A	B	C	D	E
1	3	1	1	1	0
2	0	4	5	2	2
3	4	0	5	0	7
4	5	1	0	1	1
5	5	1	0	2	5

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2) c).

	A	B	C	D	E
1	3	1	1	1	0
2	0	4	5	2	0
3	4	0	5	0	7
4	5	1	0	1	1
5	5	1	0	2	5

lowest number = 1 . ∴ Plus 1 to all values covered by two lines, subtract 1 from all uncovered values.

	A	B	C	D	E
1	3	0	1	0	0
2	0	3	5	1	2
3	5	0	6	0	8
4	5	0	0	0	1
5	5	0	0	1	5

~~Not a line~~.

	A	B	C	D	E
1	3	0	1	0	0
2	0	3	5	1	2
3	5	0	6	0	8
4	5	0	0	0	1
5	5	0	0	1	5

Five lines used to cover all zeros.

d). Alice → Game 2
Baji → Game 3
Cath → Game 5
Dip → Game 4
Ede → Game 1

B2

M

e) 20 + 17 + 20 + 15 + 20 = 92.
Maximum total score = 92.

Commentary

(a) The explanation is similar to that from many who did not understand why the $20–x$ transformation of variable was being used. It was necessary to comment on the fact that the Hungarian Algorithm is used to minimise total scores and that individual entries would give an indication of points **not** scored when the values are subtracted from twenty. (b) This candidate scores full marks for reducing by columns then rows. It is clear that the printed answer helped many to be successful here. (c) The algorithm is applied correctly and the various lines covering the zeros are clearly marked so that full marks are scored here also. (d) A common error was only giving a single matching from the table when there are actually 3 different pairings of people to games that maximise the score. (e) The maximum total score is found correctly.

Mark Scheme

Q	Solution	Marks	Total	Comments
2(a)	Hungarian algorithm minimises	E1		
	$20 - x$ indicates how many points NOT scored	E1	2	idea of high becoming low
(b)	3 4 1 3 0 0 7 5 4 2 4 3 5 2 7 7 6 2 5 3 5 4 0 4 5	B1		
	3 1 1 1 0 0 4 5 2 2 4 0 5 0 7 7 3 2 3 3 5 1 0 2 5	M1		column reduction, allow one slip from $20 - x$ table
	3 1 1 1 0 0 4 5 2 2 4 0 5 0 7 5 1 0 1 1 5 1 0 2 5	A1	3	then row reduction AG but previous table must be correct
(c)	Lines drawn	B1		4 0 5 0 7
	Reduce all uncovered by 1 and add 1 to all doubly covered	M1		
	3 0 1 0 0 0 3 5 1 2 5 0 6 0 8 5 0 0 0 1 5 0 0 1 5	A1	3	allow M1A1 if lines not as above
(d)	Choosing zeros in first and last columns Alice – Game 2; Ede – Game 1	B1		Allow if only circles around these entries with no matching listed
	Possible options B – 3 ; D – 4 ; C – 5 B – 4 ; D – 3 ; C – 5 B – 5 ; C – 4 ; D – 3	 B1 B1 B1	 4	
(e)	Maximum score = 92	B1	1	
	Total		**13**	

Question 3

3 Two people, Roseanne and Collette, play a zero-sum game. The game is represented by the following pay-off matrix for Roseanne.

		Collette		
	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
Roseanne	$\mathbf{R_1}$	-3	2	3
	$\mathbf{R_2}$	2	-1	-4

(a) (i) Find the optimal mixed strategy for Roseanne. *(7 marks)*

(ii) Show that the value of the game is -0.5. *(1 mark)*

(b) (i) Collette plays strategy C_1 with probability p and strategy C_2 with probability q. Write down, in terms of p and q, the probability that she plays strategy C_3. *(1 mark)*

(ii) Hence, given that the value of the game is -0.5, find the optimal mixed strategy for Collette. *(4 marks)*

Student Response

3.a) i) let R_1 be ae probability, p

let R_2 be ae probability, $1-p$

if Collette chooses C_1:

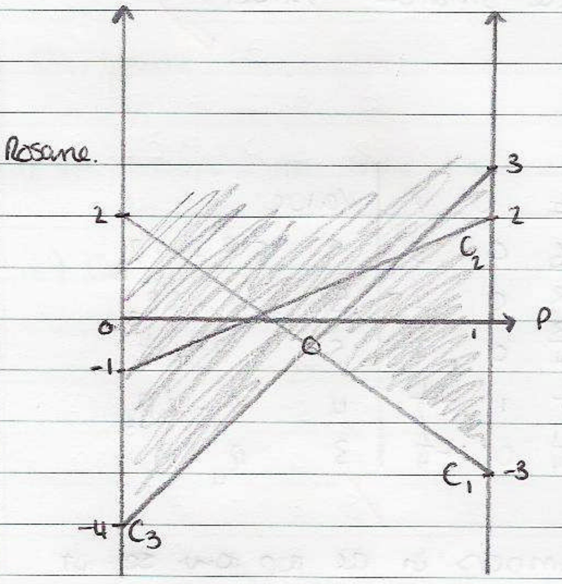
Pay-off for Rosanne $\Rightarrow$ $-3p+2(1-p) = -3p+2-2p = 2-5p$

if Coll. chooses C_2:

Pay-off for Ros. $\Rightarrow$ $2p-1(1-p) = 2p-1+p = 3p-1$

if Coll. chooses C_3:

pay-off for Ros. $\Rightarrow 3p - 4(1-p) = 3p - 4 + 4p = 7p - 4$



$C_1 = C_3$

$\Rightarrow 2 - 5p = 7p - 4$

$6 = 12p$

$\therefore p = \frac{1}{2}$

Rosanne plays R_1 with probability $= \frac{1}{2}$

Rosanne plays R_2 with probability $= \frac{1}{2}$

3.a) ii) $3(\frac{1}{2}) - 1 = 0.5$ Wrong equation B0

3.b) i) ~~3p + 4q - 3~~ $-3p + 4q$ ✗

3.b) ii) ~~4~~ $-3p + 4q = -0.5$ M0 $\Rightarrow$ Collette plays C_1 with probability $= \frac{3}{4}$

$-3(\frac{1}{2}) + 4q = -0.5$

$-1.5 + 4q = -0.5$ $\Rightarrow$ Collette plays C_2 with probability $= \frac{1}{4}$

$4q = 1$

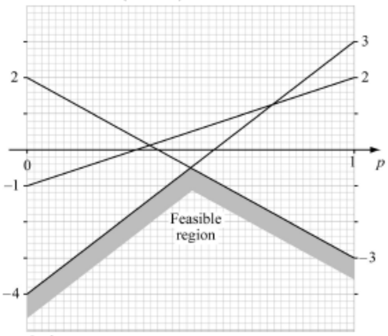
$\therefore q = \frac{1}{4}$

Commentary

(a)(i) It is a good idea to explain what p represents before writing down expressions. A better statement might have been that "Roseanne plays R_1 with probability p", but what the candidate writes here, although badly worded, is understood. The expected values when Collette chooses each of the columns are calculated correctly. The diagram is a good example for students to copy, because the values when $p = 0$ and $p = 1$ are very clear and the lines are labelled to allow the correct pair of expressions to be chosen and equated. Having found that $p = \frac{1}{2}$, the optimal mixed strategy for Roseanne is explained in words.

Many candidates did not write such a statement and lost a mark. (ii) Instead of using either of the two expressions used previously to show that the value of the game is -0.5, the candidate chooses to substitute $p = \frac{1}{2}$ into the third expression and therefore loses the mark for this part. (b) Most candidates scored a mark for getting $1-p-q$ for the probability that Collette played strategy C_3, but this candidate wrote down the wrong expression in p and q and made no progress with the rest of the question.

Mark Scheme

Q	Solution	Marks	Total	Comments
3(a)(i)	Roseanne plays R_1 with prob p Expected value when Collette plays $C_1: -3p+2(1-p)=2-5p$			
	$C_2:\ 2p-(1-p)=3p-1$	M1		One correct unsimplified
	$C_3:\ 3p-4(1-p)=7p-4$	A1		All correct unsimplified
		M1 A1		drawing 'their' lines (2 'correct' ft) correct with values clear at $p = 0$ and $p = 1$
	Solving $2-5p=7p-4$ $6=12p$	M1		their highest point; SC B1 if $p=\frac{1}{2}$ found from graph
	$\Rightarrow p=\frac{1}{2}$	A1		
	Strategy is to play R_1 for 50% of time	E1√	7	
(ii)	Value $=2-5\left(\frac{1}{2}\right)$ or $7\left(\frac{1}{2}\right)-4=-\frac{1}{2}$	B1	1	AG CSO $p=\frac{1}{2}$ and both expressions correct
(b)(i)	Let Collette play C_1 with prob p and C_2 with prob q $\Rightarrow C_3$ with prob $1-p-q$	B1	1	
(ii)	$-3p+2q+3(1-p-q)=-\frac{1}{2}$ $2p-q-4(1-p-q)=-\frac{1}{2}$	M1		Either equation LHS correct Condone $(1-p+q)$ used
	$\Rightarrow 6p+q=3\frac{1}{2}$ $6p+3q=3\frac{1}{2}$	A1		Either equation correct and simplified p & q coefficients
	$\left.\begin{array}{l}\Rightarrow p=\frac{7}{12}\\ q=0\end{array}\right\}$	A1		CSO
	$\Rightarrow$ Collette plays C_1 with prob $\frac{7}{12}$, (never plays C_2), and plays C_3 with prob $\frac{5}{12}$	E1	4	Must have statement with C_1 & C_3 **correct only**
	Total		**13**	

Question 4

4 A linear programming problem consists of maximising an objective function P involving three variables x, y and z. Slack variables s, t, u and v are introduced and the Simplex method is used to solve the problem. Several iterations of the method lead to the following tableau.

P	x	y	z	s	t	u	v	*value*
1	0	−12	0	5	−3	0	0	37
0	1	−8	0	1	2	0	0	16
0	0	4	0	0	3	0	1	20
0	0	2	0	−3	2	1	0	14
0	0	1	1	2	5	0	0	8

(a) (i) The pivot for the next iteration is chosen from the **y-column**. State which value should be chosen and explain the reason for your choice. *(2 marks)*

(ii) Perform the next iteration of the Simplex method. *(4 marks)*

(b) Explain why your new tableau solves the original problem. *(1 mark)*

(c) State the maximum value of P and the values of x, y and z that produce this maximum value. *(2 marks)*

(d) State the values of the slack variables at the optimum point. Hence determine how many of the original inequalities still have some slack when the optimum is reached. *(2 marks)*

Student Response

4) a)

i) Pivot - $20 \div 4 = 5$.

$14 \div 2 = 7$

$8 \div 1 = 8$ ✓

The pivot is 4. because this gives a lower value than 2 or 1.

ii)

P	x	y	z	s	t	u	v	value
1	0	0	0	5	6	0	3	97 ✓ B1
0	0	0	0	1	8	0	2	56 ✗
0	0	1	0	0	$3/4$	0	$1/4$	5 ✓
0	0	0	0	-3	$1/2$	1	$-1/2$	4 ✓ B1
0	0	0	1	2	$17/4$	0	$-1/4$	3 ✓ B1

$R_3 \div 4$

$R_4 - 2R_3$

$R_2 + 8R_3$

$R_1 + 12R_3$

$R_5 - R_3$

b) The new tableau solves the problem, because there is no negative numbers in the first row therefore the problem has been solved. ✓

c) $P = 97$ ✓ B1

$x = 0$ ✗

$y = 5$ ✗ B0

$z = 3$

Commentary

(a) (i)The candidate shows the various quotients and explains why 4 is chosen as the pivot. Better candidates also mentioned that 5 was the smallest **positive** value when the various divisions had been performed.
(ii) An error occurs on the second row when performing the row operations. Candidates should realise that if a column has a non-zero entry then the column cannot become the zero vector after row operations have been carried out. The rest of the tableau is correct and the candidate copes well with the fractions. Another point of commendation is the listing of the actual row operations being performed.
(b) Almost every candidate stated a reason for the optimum having been reached – even when their first row did have negative entries!
(c) The error in the final tableau meant that the candidate could not find the value of *x* when the optimum value of P had been achieved.

Mark Scheme

Q	Solution	Marks	Total	Comments
4(a)(i)	4 is chosen as pivot $\frac{20}{4}=5<\frac{14}{2}=7$ and $5<\frac{8}{1}=8$	B1 E1	2	Must have 3 values possibly unsimplified plus comment about smallest (positive) quotient
(ii)	$\begin{array}{ccccccccc} P & x & y & z & s & t & u & v & \text{value} \\ 1 & 0 & 0 & 0 & 5 & 6 & 0 & 3 & 97 \\ 0 & 1 & 0 & 0 & 1 & 8 & 0 & 2 & 56 \\ 0 & 0 & 1 & 0 & 0 & \frac{3}{4} & 0 & \frac{1}{4} & 5 \\ 0 & 0 & 0 & 0 & -3 & \frac{1}{2} & 1 & -\frac{1}{2} & 4 \\ 0 & 0 & 0 & 1 & 2 & 4\frac{1}{4} & 0 & -\frac{1}{4} & 3 \end{array}$	B1 B1 B1 B1	4	may be left as $\{0\ 0\ 4\ 0\ 0\ 3\ 0\ 1\ 20\}$ or multiples of these rows SC MI for row operations if wrong pivot used SC B1+B1 max ft if pivot row incorrect after ÷ 4
(b)	Optimum since no negative values in first row	E1	1	Must have attempted row operations
(c)	Maximum $P=97$ $x=56,\ y=5,\ z=3$	B1√ B1√	2	
(d)	$s=0,\ t=0,\ v=0,\ u=4$ $\Rightarrow$ only 1 of original inequalities has some slack	B1√ E1√	2	Ft if >1 non-zero slack variables
	Total		**11**	

Question 5

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

A small firm produces high quality cabinets.

It can produce up to 4 cabinets each month.

Whenever at least one cabinet is made during that month, the overhead costs for that month are £300.

It is possible to hold in stock a maximum of 2 cabinets during any month.

The cost of storage is £50 per cabinet per month.

The orders for cabinets are shown in the table below. There is no stock at the beginning of January and the firm plans to clear all stock after completing the April orders.

Month	January	February	March	April
Number of cabinets required	3	3	5	2

(a) Determine the total cost of storing 2 cabinets and producing 3 cabinets in a given month. *(2 marks)*

(b) By completing the table of values on **Figure 3**, or otherwise, use dynamic programming, **working backwards from April**, to find the production schedule which minimises total costs. *(8 marks)*

(c) Each cabinet is sold for £2000 but there is an additional cost of £300 for materials to make each cabinet and £2000 per month in wages. Determine the total profit for the four-month period. *(3 marks)*

Student Response

5) a) Store 2 make 3 = $2 \times 50 + 300$ = £400

b) Insert - min total cost = £1250

c) 13 sold ⇒ 13×2000 = £26 000

13 made ⇒ 300×13 = -£3900

Wages ⇒ 4×2000 = -£8000

Min cost ⇒ = - £1250

Total = £12850

Profit of ~~£12850~~ £12 850

Month & Demand	Initial State	Action	Destination State	Value	
April	0	2	0	300 + 0 = 300	*
(demand 2)					
	1	1	0	300 + 50 = 350	*
	2	0	0	0 + 100 = 100	*
March	1	4	0	300 + 50 + 300 = 650	*
(demand 5)					
	2	3	0	100 + 300 + 300 = 700	*
		4	1	100 + 300 + 350 = 750	
February	0	4	1	300 + 650 = 950	*
(demand 3)					
	1	3	1	50 + 300 + 650 = 1000	*
		4	2	50 + 300 + 700 = 1050	
	2	2	1	100 + 300 + 650 = 1050	*
		3	2	100 + 300 + 700 = 1100	
January	0	3	0	300 + 950 = 1250	*
(demand 3)		4	1	300 + 1000 = 1300	

Production Schedule which minimises total costs

Month	January	February	March	April
Number of cabinets made	3	4	4	2

Turn over ▶

Commentary

This is a very good solution to the question demonstrating a clear understanding of dynamic programming. The initial calculation in part(a) is correct . Those who misunderstood the context multiplied £300 by 3 and therefore could not find the correct total cost.
Part (b) is done on the insert and all the relevant calculations are shown. For each month the relevant minimum values are indicated by an asterisk and these are used in the relevant calculations for the previous month. The asterisk alongside £1 250 in January signifies that 3 cabinets need to be made in January and by working backwards 4 need making in February and so on.
Many candidates obtained an answer of £14 100 for part (c) but this candidate realises the need to deduct the minimum cost of production, namely £1 250 so as to find the correct total profit of £12 850.

Mark Scheme

<table>
<tr><th>Q</th><th>Solution</th><th>Marks</th><th>Total</th><th>Comments</th></tr>
<tr><td>5(a)</td><td>Overhead cost = £300

Storing 2 cabinets $= 2 \times £50$
$\Rightarrow$ Total cost = £400</td><td>M1

A1</td><td>

2</td><td>considering overhead and storage of 2 cabinets</td></tr>
<tr><td>(b)</td><td>March values £700
£750
Choosing minima for March (at least one), their 650 or 700 seen in February values

February state 0
$300+0+650=950$

February state 1
$300+50+650 = 1000$
$300+50+700 = 1050$
February state 2
$300+100+650=1050$
$300+100+700=1100$
January values
1250 and 1300

Choosing least value of January and working backwards through table to select actions A_1, A_2 and A_3
Schedule correct</td><td>B1
B1

M1

B1

A1

B1

M1
A1</td><td>

8</td><td>
<table>
<tr><th>Month</th><th>State</th><th>Value</th><th></th></tr>
<tr><td>Apr</td><td>0</td><td>300 + 0 = 300</td><td>A_3</td></tr>
<tr><td></td><td>1</td><td>300 + 50 = 350</td><td></td></tr>
<tr><td>Mar</td><td>1</td><td>300 + 50 + 300 = 650</td><td>A_2</td></tr>
<tr><td></td><td>2</td><td>300 + 100 + 300 = 700</td><td>Min</td></tr>
<tr><td></td><td></td><td>300 + 100 + 350 = 750</td><td></td></tr>
<tr><td>Feb</td><td>0</td><td>300 + 0 + 650 = 950</td><td>A_1</td></tr>
<tr><td></td><td>1</td><td>300 + 50 + 650 = 1000</td><td>Min</td></tr>
<tr><td></td><td></td><td>300 + 50 + 700 = 1050</td><td></td></tr>
<tr><td></td><td>2</td><td>300 + 100 + 650 = 1050</td><td></td></tr>
<tr><td></td><td></td><td>300 + 100 + 700 = 1100</td><td></td></tr>
<tr><td>Jan</td><td>0</td><td>300 + 0 + 950 = 1250</td><td>Min</td></tr>
<tr><td></td><td></td><td>300 + 0 + 1000 = 1300</td><td></td></tr>
</table>

SC: B1 for schedule without DP
<table>
<tr><th>Jan</th><th>Feb</th><th>Mar</th><th>Apr</th></tr>
<tr><td>3</td><td>4</td><td>4</td><td>2</td></tr>
</table>
Should get 3 or 4 when table completed</td></tr>
<tr><td>(c)</td><td>Profit excluding answer to (b)
$13 \times £(2000 - 300)$
$- 4 \times £2000$
$= £14100$
Total profit over 4 months is
$£14100 - £1250$
$= £12850$</td><td>
M1

A1

A1√</td><td>

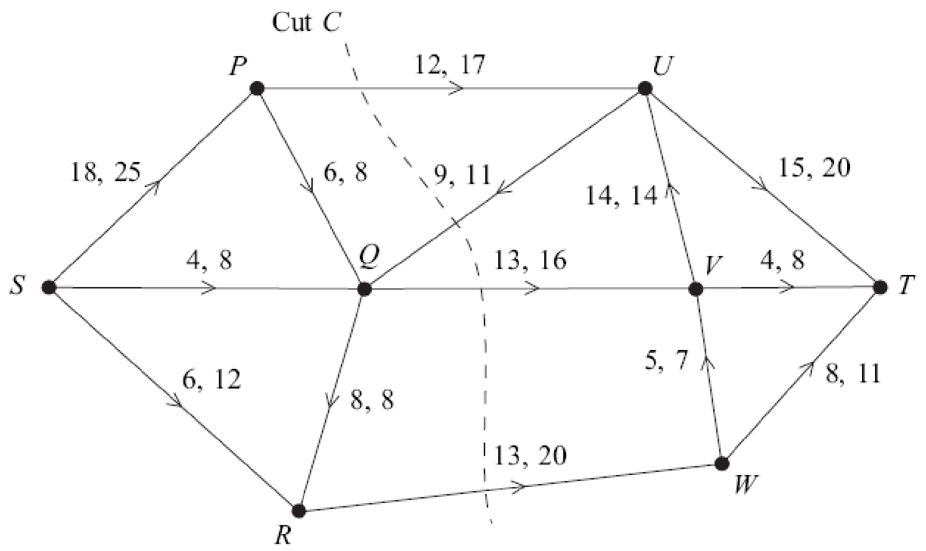
3</td><td>
Generous

Ft their £1250</td></tr>
<tr><td></td><td>Total</td><td></td><td>13</td><td></td></tr>
</table>

Question 6

6 [**Figures 4**, **5 and 6**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) (i) Find the value of the cut C. *(1 mark)*

(ii) Hence state what can be deduced about the maximum flow from S to T. *(1 mark)*

(b) **Figure 4**, printed on the insert, shows a partially completed diagram for a feasible flow of 32 litres per second from S to T. Indicate, on **Figure 4**, the flows along the edges PQ, UQ and UT. *(3 marks)*

(c) (i) Taking your feasible flow from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 5**. *(2 marks)*

(ii) Use flow augmentation on **Figure 5** to find the maximum flow from S to T. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

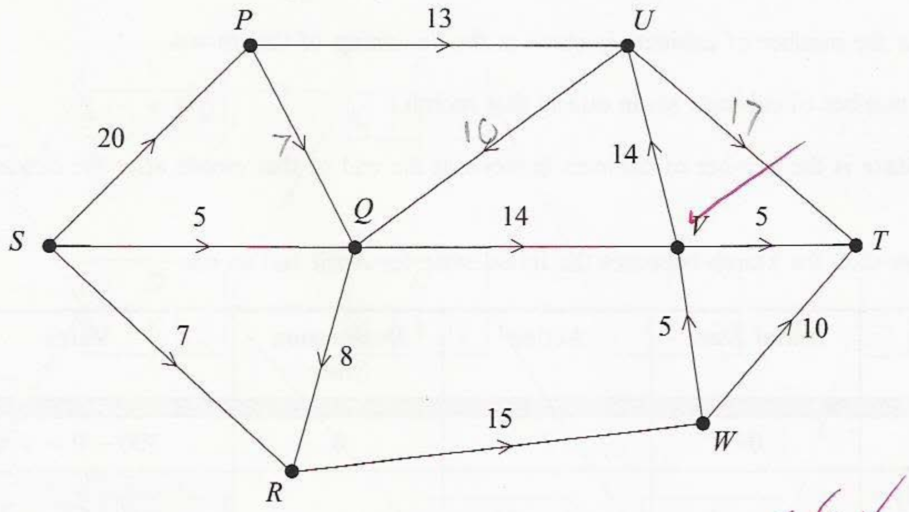
(iii) Illustrate the maximum flow on **Figure 6**. *(1 mark)*

Student Response

6) a) i) $C = 17 - 9 + 16 + 20$
$= 44$ ✓ B1

ii) maximum flow is less than or equal to 44 ✓ B1

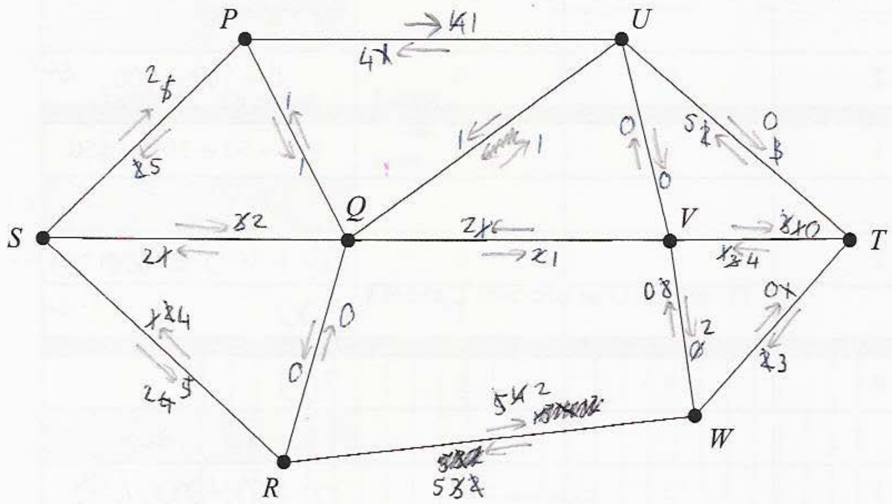
Figure 4 (for use in Question 6)



3

IF ✓ 2

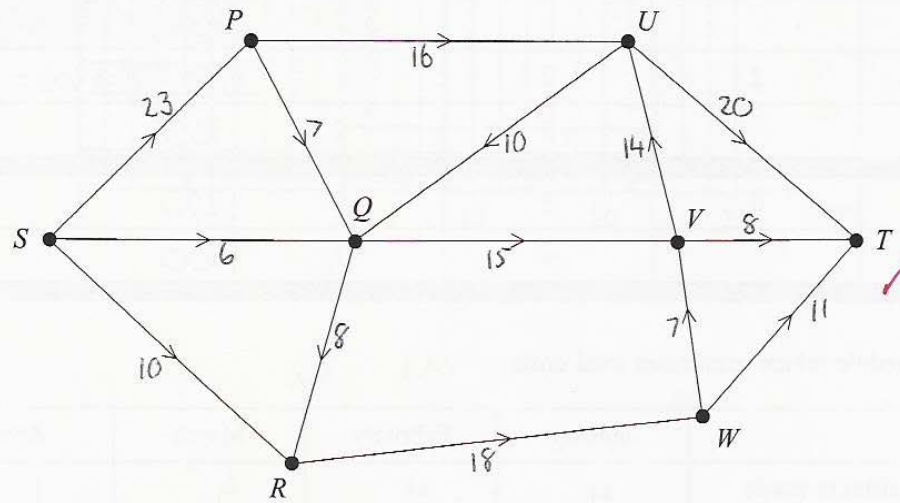
Figure 5 (for use in Question 6)



Path	Additional Flow
SPUT	3
SRWT	1
SRWVT	2
SQVT	1

5

Figure 6 (for use in Question 6)



1

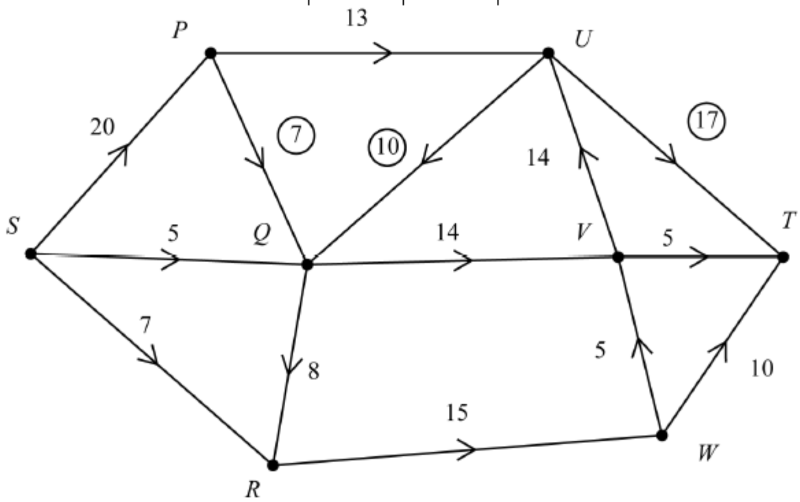
1

Commentary

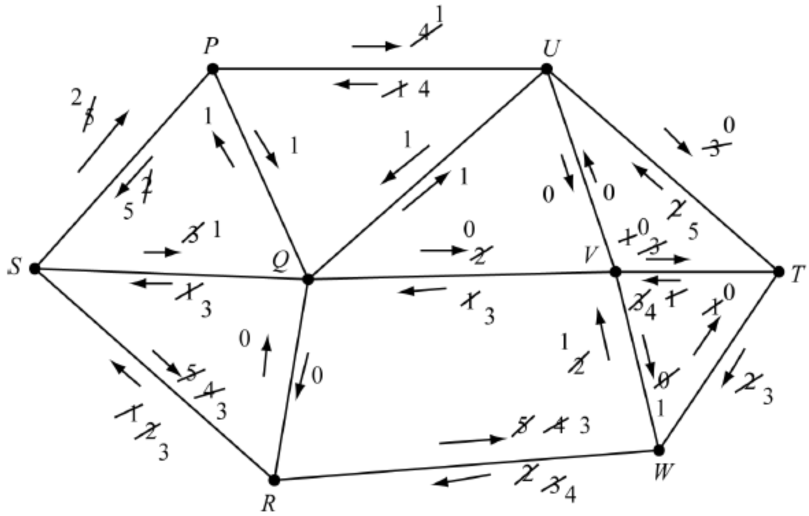
This is a good response to this question. The value of the cut is calculated correctly and the correct statement made about the maximum flow. On Figure 4, the correct values of the flows along the edges PQ, UQ and UT are found and used to produce an initial flow on Figure 5. These are indicated in ink and when the flow is adjusted it is easy to see both the new and old figures on the network. The solution is slightly different from that in the mark scheme and in fact there were lots of possible flow diagrams giving a correct maximum flow of 39. This solution illustrates that it is possible to present a solution where all the adjustments are legible and can be given full credit. Many candidates would do well to copy this exemplar.

Mark Scheme

Q	Solution	Marks	Total	Comments
6(a)(i)	17 – 9 + 16 + 20 = 44	B1	1	
(ii)	Max flow ≤ 44	B1√	1	
(b)		B1		7
		B1		10
		B1	3	17

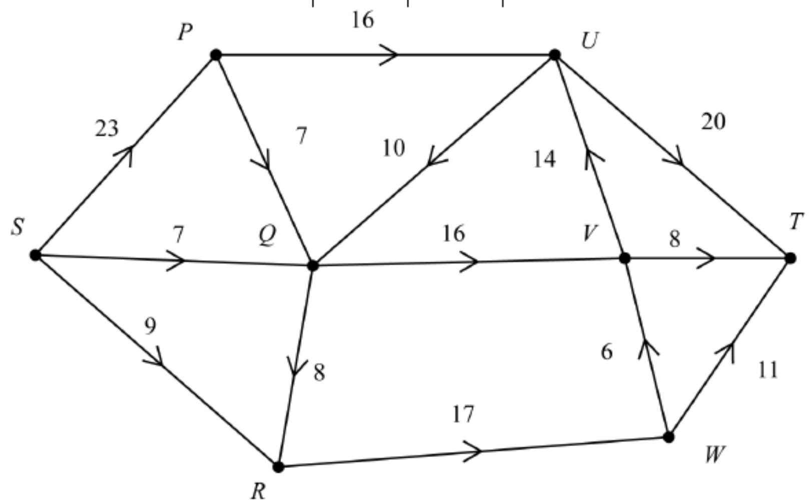


Q	Solution	Marks	Total	Comments
(c)(i)	Initial forward and backward flows	M1		5 pairs correct
	Correct	A1	2	



Q	Solution	Marks	Total	Comments
(ii)	(see table below)	M1		adjusting flows on network (1 path shown correctly)
		A1		correct
		M1		additional flow in table
		A1		second flow
		A1	5	all correct

Path	Additional Flow
SPUT	3
SQVT	2
SRWT	1
SRWVT	1

Q	Solution	Marks	Total	Comments
6(c)(iii)				
	Max flow of 39 (several possibilities of final flow diagram)	B1	1	
	Total		**13**	
	TOTAL		**75**	