



Teacher Support Materials 2008

Maths GCE

Paper Reference MPC3

Copyright © 2008 AQA and its licensors. All rights reserved.

Permission to reproduce all copyrighted material has been applied for. In some cases, efforts to contact copyright holders have been unsuccessful and AQA will be happy to rectify any omissions if notified.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334). Registered address: AQA, Devas Street, Manchester M15 6EX.
Dr Michael Cresswell, Director General.

Question 1c

1 Find $\frac{dy}{dx}$ when:

(a) $y = (3x + 1)^5$; (2 marks)

(b) $y = \ln(3x + 1)$; (2 marks)

(c) $y = (3x + 1)^5 \ln(3x + 1)$. (3 marks)

Student Response

c) $y = (3x + 1)^5 \cdot \ln(3x + 1)$

$$\frac{dy}{dx} = (3x+1)^5 \cdot \frac{3}{3x+1} + \ln(3x+1) \cdot 15(3x+1)^4$$

$$= 3(3x+1)^4 + 15 \ln(3x+1) (3x+1)^5$$

$$\frac{dy}{dx} = (9x+3)^4 + 75 \ln(3x+1)$$

2
6

Commentary

Candidates will be penalised for incorrect algebra.

This candidate correctly answered parts (a) and (b), then applied the product rule correctly to obtain the correct answer. Unfortunately the candidate proceeded to simplify their answer and produced some very poor work. They have taken an algebraic factor inside the ln expression.

The candidate was penalised the final accuracy mark.

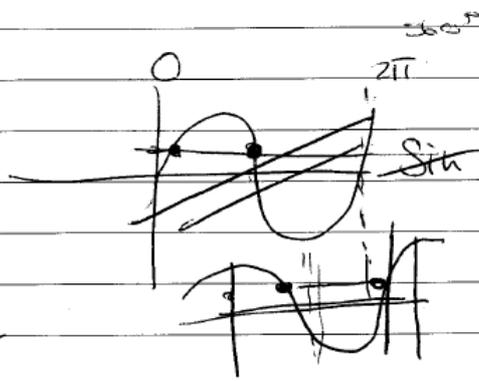
Mark scheme

<p>(c) $\frac{dy}{dx} = (3x+1)^5 \times \frac{3}{3x+1} + \ln(3x+1) \times 15(3x+1)^4$</p> $\left(= (3x+1)^4 [3 + 15 \ln(3x+1)] \right)$ $\left(= 3(3x+1)^4 [1 + 5 \ln(3x+1)] \right)$	<p>M1 A1 A1</p>	<p>3</p>	<p>product rule $uv' + u'v$ (from (a) and (b)) either term correct CSO with no further errors</p>
--	-------------------------	----------	--

Question 2

- 2 (a) Solve the equation $\sec x = 3$, giving the values of x in radians to two decimal places in the interval $0 \leq x < 2\pi$. *(3 marks)*
- (b) Show that the equation $\tan^2 x = 2 \sec x + 2$ can be written as $\sec^2 x - 2 \sec x - 3 = 0$. *(2 marks)*
- (c) Solve the equation $\tan^2 x = 2 \sec x + 2$, giving the values of x in radians to two decimal places in the interval $0 \leq x < 2\pi$. *(4 marks)*

Student response

	<p>2a) $\sec x = 3$ $0 - 2\pi$</p> <p>$\sec x = 3$ $\frac{1}{\cos x} = 3$ $\cos x = \frac{1}{3}$ 0.999 1.236° 4.372552071 $= 1.23^\circ, 4.37^\circ$</p> 	<p>Leave blank</p> <p>2</p>
	<p>b) $\tan^2 x = 2 \sec x + 2$ \tan $\sec^2 x - 2 \sec x = 3$</p>	<p>0</p>
	<p>c) $(\sec x + 1)(\sec x - 3) = 0$ 31</p> <p>$\sec x + 1 = 0$ $\sec x = -1$</p> <p>$\sec x - 3 = 0$ $\sec x = +3$</p>	

$\cos(50)$

Question number

Leave blank

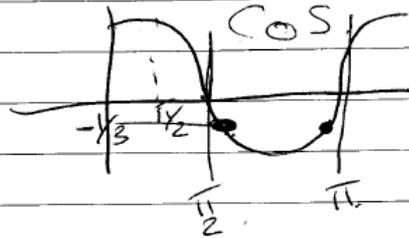
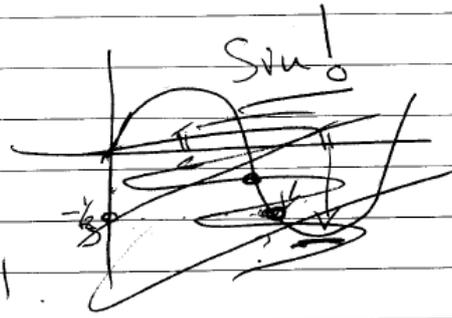
~~const.~~ $\sec x = 1$

$$\frac{1}{\cos x} = -1$$

$$\frac{1}{-1} = \cos x$$

$$\cos x = -1$$

$$x = \pi$$



When $\sec x = +3$

$$\frac{1}{\cos x} = +3$$

$$\Rightarrow \frac{1}{3} = \cos x$$

~~$x =$~~

~~$$\cos^{-1} x = 1.910$$

$$+ \pi = 5.0522$$~~

~~$$x = 1.91^\circ$$~~

~~$$x = 5.05^\circ$$~~

4

$$\frac{1}{\cos x} = 3$$

$$\frac{1}{3} = \cos$$

$$\sqrt{\quad} = 1.230959$$

$$\sqrt{\quad} = 4.37255$$

6

$$x = 1.23^\circ, 4.37^\circ$$

Commentary

Candidates sometimes make mistakes under exam pressure but then maintain that 'mistake'.

If that is the case they will, normally, only be penalised once in the question.

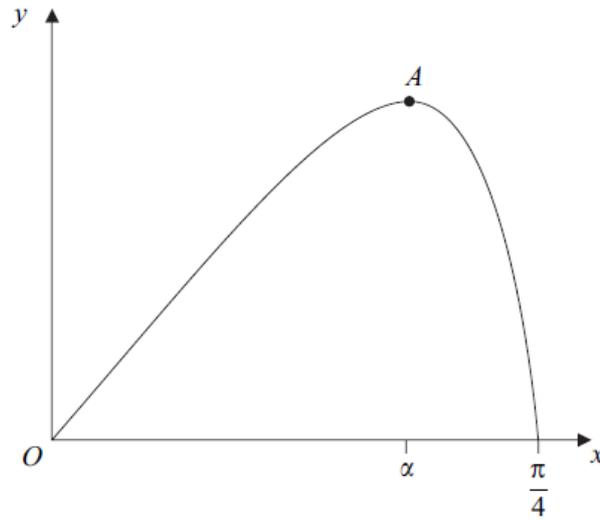
This candidate made a mistake in part (a) of the question but they 'carried forward' their answers to part (c) and this was then recorded. They did lose extra marks in this part through not solving the second part of the quadratic equation.

Mark Scheme

2(a)	$x = \cos^{-1} \frac{1}{3}$ $= 1.23, 5.05 \quad (0.39\pi, 1.61\pi)$	M1 A1,A1	3	PI AWRT (-1 for each error in range) SC 70.53, 289.47 B1
(b)	$\sec^2 x - 1 = 2\sec x + 2$ $\sec^2 x - 2\sec x - 3 = 0$	M1 A1	2	use of $\sec^2 x = 1 + \tan^2 x$ AG; CSO
(c)	$\sec^2 x - 2\sec x - 3 = 0$ $(\sec x - 3)(\sec x + 1) = 0$ $\cos x = \frac{1}{3} \text{ or } -1 \quad \text{o.e.}$ $x = 1.23, 5.05,$ $3.14 \quad (\pi)$	M1 A1 B1f B1	4	(2 answers in range from (a)) AWRT all correct and no extras in range SC 70.53, 289.47, 180 B1
Total			9	

Question 3(a)(iii)

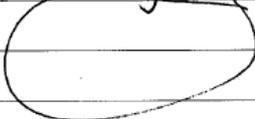
3 A curve is defined for $0 \leq x \leq \frac{\pi}{4}$ by the equation $y = x \cos 2x$, and is sketched below.



(a) Find $\frac{dy}{dx}$.

(2 marks)

Student Response

Question number	Leave blank
3) a)	
$y = x \cos 2x$ $\frac{dy}{dx} = \cancel{2x} x - 2 \sin 2x + \cos 2x$ $= \cancel{2x \sin 2x} - 2x \sin 2x + \cos 2x$	2
i) $1 - 2x \tan 2x = 0$ \times $\tan 2x = \frac{1}{2}$ $+ 2x \tan 2x = +1$	0
ii) $f(x) = x \cos 2x$ $f(x) = 1 - 2x \tan 2x$ $f(0.4) = 0.2786$ $f(0.4) = 0.17628$ $f(0.5) = -0.5574$	1
<p>as there is a change of sign it lies between the points</p> 	
iii) $1 - 2x \tan 2x = 0$ $-2x \tan 2x = -1$ $x \tan 2x = \frac{+1}{+2}$	0
$x \tan 2x = \frac{1}{2} \times \frac{1}{\tan 2x}$ $= \frac{1}{2} \times \left(\frac{1}{\tan} \times \frac{1}{2x} \right)$ \times $= \frac{1}{2} \tan^{-1} \left(\frac{1}{2x} \right)$	

Commentary

On this paper candidates will have parts of questions where the answer will be given and candidates have to show the result.

Examiners check working carefully.

Regularly candidates will make major blunders in a solution and then 'miraculously' arrive at the correct answer.

These candidates as in the shown example will not score the marks.

Mark Scheme

Q	Solution	Marks	Total	Comments
3(a)	$\frac{dy}{dx} = -x^2 \sin 2x + \cos 2x$	M1 A1	2	product rule $kx \sin 2x \pm \cos 2x$ no further incorrect working

Question 3(c)

(c) Use integration by parts to find $\int_0^{0.5} x \cos 2x \, dx$, giving your answer to three significant figures. (5 marks)

Student Response

Question number	Response	Leave blank
	$\text{iv. when } n=1, x=2: \therefore x_2 = \frac{1}{2} \tan^{-1}\left(\frac{1}{2 \times 0.4}\right) = \underline{0.45}$	
	$\text{when } n=2, x=3: \therefore x_3 = \frac{1}{2} \tan^{-1}\left(\frac{1}{2 \times 0.45}\right) = \underline{0.42}$	2
c)	$\int_0^{0.5} x \cos 2x \, dx \Rightarrow u = x, \quad v = \frac{\sin 2x}{2}$ $\frac{du}{dx} = 1$ $\therefore \int_0^{0.5} x \cos 2x \, dx = uv - \int v \frac{du}{dx} \cdot dx$ $= \frac{x \sin 2x}{2} - \int \frac{\sin 2x}{2} \cdot 1 \, dx$ $= \left[\frac{x \sin 2x}{2} + \frac{\cos 2x}{4} \right]_0^{0.5}$ $= \left[\frac{0.5 \sin 1}{2} + \frac{\cos 1}{4} \right] - \left[0 + \frac{1}{4} \right]$ $= \frac{0.8415}{4} + \frac{0.5403}{4} - \frac{1}{4}$ $= 0.2104 + 0.1351 - 0.25$ $= \underline{0.096}$	4 (13)

Commentary

In questions that require numerical evaluation candidates must ensure that they answer to the required degree of accuracy.

This candidate has correctly used integration by parts, substituted values into their expression correctly but then has written the final answer to 2 significant figures (or to 3 decimal places!) and hence have lost the final accuracy mark.

Mark Scheme

(c)	$y = x \cos 2x$ $\left. \begin{array}{l} u = x \quad du = 1 \\ dv = \cos 2x \quad v = \frac{\sin 2x}{2} \end{array} \right\}$ $\int = \frac{x \sin 2x}{2} - \int \frac{\sin 2x}{2} (dx)$ $= \left[\frac{x \sin 2x}{2} + \frac{\cos 2x}{4} \right]_{(0)}^{(0.5)}$ $= \left(\frac{\sin 1}{4} + \frac{\cos 1}{4} \right) - \left(\frac{\cos 0}{4} \right)$ $= 0.0954$	M1		differentiate one term } must be $k \sin 2x$ integrate one term }
		m1		
		A1		
		m1		correctly substituting values from previous 2 method marks
		A1	5	AWRT

Question 4(c)

4 The functions f and g are defined with their respective domains by

$$f(x) = x^2, \quad \text{for all real values of } x$$

$$g(x) = \frac{1}{2x-3}, \quad \text{for real values of } x, \quad x \neq \frac{3}{2}$$

- (a) State the range of f . (1 mark)
- (b) (i) The inverse of g is g^{-1} . Find $g^{-1}(x)$. (3 marks)
- (ii) State the range of g^{-1} . (1 mark)
- (c) Solve the equation $fg(x) = 9$. (3 marks)

Question number

Leave blank

$$2x = \frac{1 + 3y}{y}$$

$$x = \frac{1}{2} \times \frac{1 + 3y}{y}$$

$$x = \frac{1 + 3y}{2y}$$

$$g(x) = \frac{1 + 3x}{2x}$$

ii) $g(x) = 0$

$$g(x) \geq 1$$

c) $f(x) = 9$

$$f(x) = \left(\frac{1}{2x-3}\right)^2$$

$$9 = \left(\frac{1}{2x-3}\right)^2$$

$$9 = \frac{1}{4x^2 - 12x + 9}$$

$$9(4x^2 - 12x + 9) = 1$$

$$36x^2 - 108x + 81 = 1$$

$$36x^2 - 108x + 80 = 0$$

$$a = 36$$

$$b = -108$$

$$c = 80$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{108 \pm \sqrt{11664 - 4 \times 36 \times 80}}{72}$$

$$= \frac{108 \pm \sqrt{144}}{72}$$

Question number

Let blank

$$x = \frac{100 \pm \sqrt{144}}{72}$$

$$x = \frac{100 \pm 12}{72}$$

$$x = \frac{120}{72} \quad x = \frac{96}{72}$$

$$\underline{\underline{x = \frac{2}{3}}} \quad \underline{\underline{x = \frac{1}{3}}}$$

Commentary

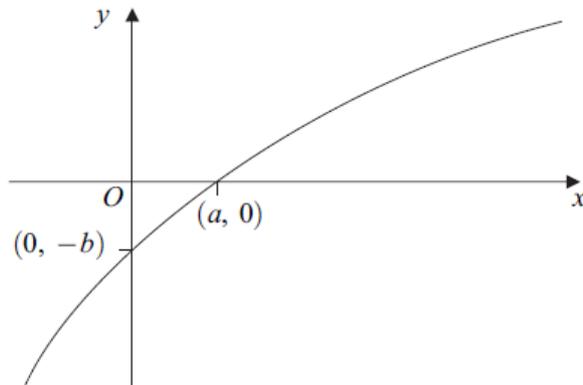
Although the mark scheme solved this part of the question using simple modulus functions it was possible to solve the question by using a quadratic equation. This candidate has produced a perfect solution. The working has taken more than the other printed solution but this candidate was able to solve the quadratic carefully. A candidate could have scored full marks if they arrived at the correct quadratic equation and then written down the answers using a graphical calculator.

Mark Scheme

(c)	$\left(\frac{1}{2x-3}\right)^2 = 9$	B1		square root and invert (condone missing \pm) alternative: attempt to solve a quadratic that comes from $4x^2 - 12 + 9 = \frac{1}{9}$ o.e.
	$2x-3 = \pm \frac{1}{3}$	M1		
	$x = \frac{5}{3}, \frac{4}{3}$	A1	3	

Question 5(a)

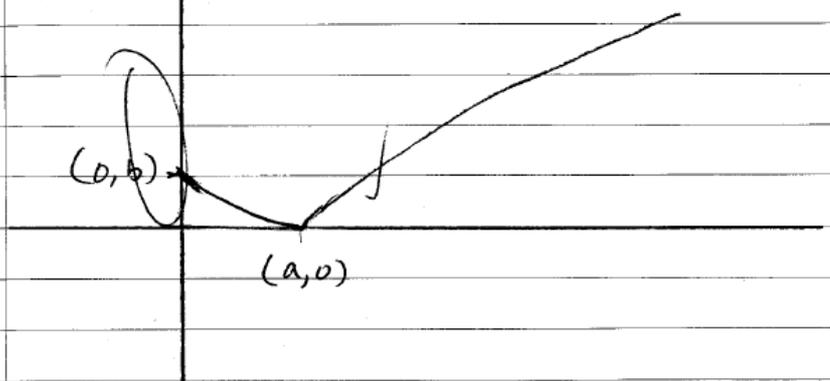
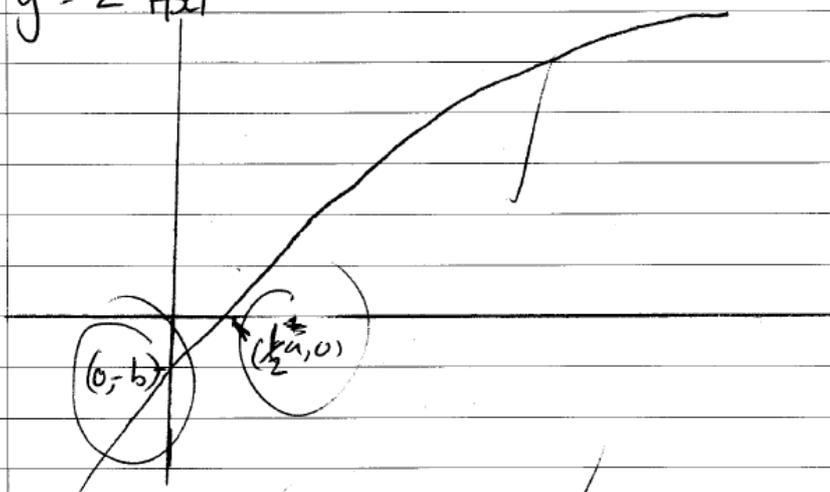
- 5 (a) The diagram shows part of the curve with equation $y = f(x)$. The curve crosses the x -axis at the point $(a, 0)$ and the y -axis at the point $(0, -b)$.



On separate diagrams, sketch the curves with the following equations. On each diagram, indicate, in terms of a or b , the coordinates of the points where the curve crosses the coordinate axes.

- (i) $y = |f(x)|$. *(2 marks)*
- (ii) $y = 2f(x)$. *(2 marks)*

Student Response

<p>5a) $y = f(x)$</p> 	Leave blank
<p>$y = 2 f(x)$</p> 	
<p>bi) transformation of vector $\begin{bmatrix} -1 \\ -2 \end{bmatrix}$ ✓</p> <p>stretch on the y axis of scale factor 4</p>	6

Commentary

This question was poorly answered with the majority of candidates unaware of the modulus function.

The first sketch initially followed the correct shape but the candidate obviously believed that a modulus function meant that 'y' could not be negative. The candidate still scored 1 mark for the correct coordinates on the axes.

The second sketch is the correct shape but only scores 1 mark as the intersections on the axes are incorrect.

The transformation has been incorrectly interpreted.

Mark Scheme

<p>5(a)(i)</p>		<p>B1 B1</p>	<p>2</p>	<p>shape coordinates</p>
<p>(ii)</p>		<p>B1 B1</p>	<p>2</p>	<p>shape coordinates</p>

Question 5(b)(i)

(b) (i) Describe a sequence of geometrical transformations that maps the graph of $y = \ln x$ onto the graph of $y = 4 \ln(x + 1) - 2$. (6 marks)

Student Response

bi) $y = \ln x \rightarrow y = 4 \ln(x+1) - 2$.

translation $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ with a stretch in y direction s/f 4.

3

Commentary

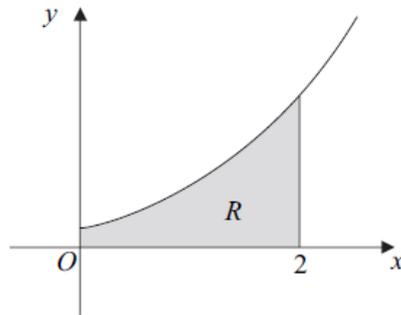
Candidates must 'think through' graphical transformations. The function needed the translation to be split into 2 distinct parts either side of the stretch Or to adjust the original function first. The order of transformations is vital.

Mark Scheme

(b)(i)	Translation	M1			OR I stretch M1 I + (II or III) II SF 4 III // y-axis A1 (I + II + III) Translation M1
	$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$	A1			
	Stretch I SF 4 II // y-axis III	M1		I + (II or III)	
	Translation $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$	A1		I + II + III	$\begin{bmatrix} -1 \\ -2 \end{bmatrix}$ A1 B1
	All correct and no mistakes on order etc	B1		both	
	Alternative: $y = 4 \ln(x+1) - 2 = 4 \left[\ln(x+1) - \frac{1}{2} \right]$	A1	6		All correct A1
	Translation	(B1)			
	$\begin{bmatrix} -1 \\ -\frac{1}{2} \end{bmatrix}$	(M1)			
	Stretch I	(A1)			
	SF 4 II	(M1)		I + (II or III)	
	// y-axis III	(A1)		I + II + III	
	All correct and no mistakes on order etc	(A1)	(6)		

Question 6(b)

6 The diagram shows the curve with equation $y = (e^{3x} + 1)^{\frac{1}{2}}$ for $x \geq 0$.



(a) Find the gradient of the curve $y = (e^{3x} + 1)^{\frac{1}{2}}$ at the point where $x = \ln 2$. (5 marks)

(b) Use the mid-ordinate rule with four strips to find an estimate for $\int_0^2 (e^{3x} + 1)^{\frac{1}{2}} dx$, giving your answer to three significant figures. (4 marks)

(c) The shaded region R is bounded by the curve, the lines $x = 0$, $x = 2$ and the x -axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° about the x -axis. (4 marks)

Student Response

6	<p>b) $\int_0^2 (e^{3x} + 1)^{1/2} dx$</p> <p>$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$</p> <p>mid-ordinates</p> <p>$x_0 = 0$</p> <p>$\rightarrow x_{0\frac{1}{2}} = 0.25 \rightarrow y_{\frac{1}{2}} = 1.77$</p> <p>$x_1 = 0.5$</p> <p>$\rightarrow x_{1\frac{1}{2}} = 0.75 \rightarrow y_{\frac{3}{2}} = 3.24$</p> <p>$x_2 = 1$</p> <p>$\rightarrow x_{2\frac{1}{2}} = 1.25 \rightarrow y_{\frac{5}{2}} = \del{6.59} 6.89$</p> <p>$x_3 = 1.5$</p> <p>$\rightarrow x_{3\frac{1}{2}} = 1.75 \rightarrow y_{\frac{7}{2}} = 13.80$</p> <p>$x_4 = 2$</p> <p>$x_{4\frac{1}{2}} =$</p> <p>$\int_0^2 (e^{3x} + 1)^{1/2} dx = 0.5 (1.77 + 3.24 + \del{6.59} + 13.8) =$</p> <p>$= 12.7 \del{7} \del{7} \underline{\underline{12.7}} \text{ (2sf)}$</p>	Leave blank
---	---	-------------

Commentary

Numerical method questions need students to carefully study the required degree of accuracy and then work to a greater degree of accuracy.

This script has worked to 3 significant figures, but then the candidate cannot be sure that the final answer is correct to 3 significant figures.

If the student had listed the functions not evaluated, they wouldn't have been penalised.

Mark Scheme

(b)	x	y	P.I				
	0.25	1.765(5)				B1	correct x values
	0.75	3.238(5)				B1	3 or 4 correct y values 4 s.f. or better
	1.25	6.597(1)				M1	
	1.75	13.84(1)				A1	4
$\int = 0.5 \times \sum y$				sc 12.7 with no working $\frac{2}{4}$			
= 12.7							

Question 6(c)

(c) The shaded region R is bounded by the curve, the lines $x = 0$, $x = 2$ and the x -axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° about the x -axis. (4 marks)

Student Response

c) $x=0$ $x=2$	
$V = \int_0^2 \pi y^2$ $y = (e^{3x+1})^{\frac{1}{2}}$ $y' = e^{3x+1}$	
$V = \int_0^2 \pi (e^{3x+1})$	
$V = \pi \int_0^2 e^{3x+1}$	
$V = \pi \int_0^2 \frac{1}{3} e^{3x+1}$	
$V = \pi \left[\frac{1}{3} e^{3x+1} \right]_0^2$	
$V = \pi \left[\frac{1}{3} e^6 + 2 \right] - \left[\frac{1}{3} e^1 + 0 \right]$	3
$V = \pi (136.47) - \frac{1}{3}$	8
$V = \underline{\underline{420.4}}$	

Commentary

Numerical method questions need students to carefully study the required degree of accuracy.

This particular question required an exact answer.

The candidate arrived at the correct expression after substituting in the limits but then failed to tidy up the expression exactly and instead resorted to the calculator for a final answer.

Mark Scheme

(c)	$v = \pi \int y^2 dx$ $= (\pi) \int (e^{3x} + 1) (dx)$ $= (\pi) \left[\frac{1}{3} e^{3x} + x \right]_{(0)}^{(2)}$ $= (\pi) \left[\left(\frac{1}{3} e^6 + 2 \right) - \left(\frac{1}{3} e^0 + 0 \right) \right]$ $= \pi \left[\frac{1}{3} e^6 + \frac{5}{3} \right]$ $\left(= \frac{\pi}{3} (e^6 + 5) \right)$	M1		
		A1		$ke^{3x} + x$
		m1		correct substitution into f ($\int e^{3x}$)
		A1	4	CSO

Question 7(c)

7 (a) Given that $y = \frac{\sin \theta}{\cos \theta}$, use the quotient rule to show that $\frac{dy}{d\theta} = \sec^2 \theta$. (3 marks)

(b) Given that $x = \sin \theta$, show that $\frac{x}{\sqrt{1-x^2}} = \tan \theta$. (2 marks)

(c) Use the substitution $x = \sin \theta$ to find $\int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx$, giving your answer in terms of x . (5 marks)

Student Response

$$7c \int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx \quad \begin{array}{l} x = \sin \theta \\ \frac{dx}{d\theta} = \cos \theta \end{array}$$

$$\int \frac{1}{(1-\sin^2 \theta)^{\frac{3}{2}}} \times \cos \theta \, d\theta$$

~~$$= \int (1-\cos^2 \theta)(\cos \theta) \, d\theta$$~~

~~$$= \int \cos^2 \theta \times \cos^2 \theta \, d\theta$$~~

~~$$= \sin \theta$$~~

$$= \int \frac{1}{(\cos^2 \theta)^{\frac{3}{2}}} \times \cos \theta \, d\theta$$

~~$$= \int \sin^2 \theta \times \cos \theta \, d\theta$$~~

$$= \int \frac{\cos \theta}{\cos^3 \theta} \, d\theta = \int \frac{1}{\cos^2 \theta} \, d\theta$$

$$= \int \sec^2 \theta \, d\theta = \underline{\underline{\tan \theta}}$$

		Leave blank
	$\tan \theta = \frac{\sin \theta}{\cos \theta}$	
	if $x = \sin \theta$ $x^2 = \sin^2 \theta$	
	$x = \sin \theta$ $x^2 = 1 - \cos^2 \theta$	
	$\tan \theta = \frac{\sin \theta}{\cos \theta}$	
	$x^2 = 1 - \cos^2 \theta$	
	$x^2 - 1 = -\cos^2 \theta$	
	$\cos^2 \theta = 1 - x^2$	
	$\cos \theta = \sqrt{1 - x^2}$	
	$\therefore = \frac{x}{\sqrt{1 - x^2}}$ ()	can't do it
		see answer to part (b)!
		4
		9

Commentary

Questions that are split into a number of parts often are linked. This final part of the question brought together work in parts (b) and (c). This candidate didn't study the question and started the final part from scratch. After solving the question they realised their 'error' as shown by their comment. Questions are normally structured and candidates should be aware of this.

Mark Scheme

(c)	$\int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx$ $x = \sin \theta$ $dx = \cos \theta d\theta$ <p style="text-align: right;">o.e.</p> $\int = \int \frac{\cos \theta (d\theta)}{(1-\sin^2 \theta)^{\frac{3}{2}}}$ $= \int \frac{\cos \theta}{(\cos^2 \theta)^{\frac{3}{2}}} (d\theta)$ $= \int \sec^2 \theta (d\theta)$ $= \tan \theta$ $= \frac{x}{\sqrt{1-x^2}} (+c)$	<p>M1</p> <p>m1</p> <p>A1</p> <p>A1</p> <p>A1</p>	<p>5</p>	$\frac{dx}{d\theta} = \pm \cos \theta$ <p>all in terms of θ</p> <p>CSO including $d\theta$'s</p>
-----	---	---	----------	---