# Teacher Support Materials
# 2008

# Maths GCE

# Paper Reference MPC1



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*Dr Michael Cresswell*, Director General.

### *Question 1*

**1** The straight line $L$ has equation $y = 3x - 1$ and the curve $C$ has equation

$$y = (x+3)(x-1)$$

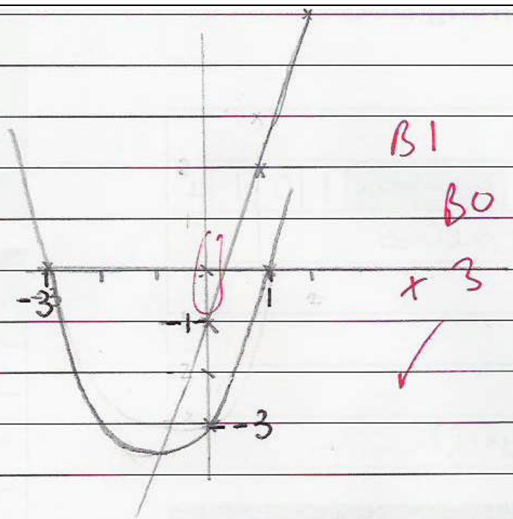
(a) Sketch on the same axes the line $L$ and the curve $C$, showing the values of the intercepts on the $x$-axis and the $y$-axis. *(5 marks)*

(b) Show that the $x$-coordinates of the points of intersection of $L$ and $C$ satisfy the equation $x^2 - x - 2 = 0$. *(2 marks)*

(c) Hence find the coordinates of the points of intersection of $L$ and $C$. *(4 marks)*

### *Student Response*

1a)




B1

B0

× 3

$y = 3x - 1$

$y = (x+3)(x-1)$

$y = x^2 + 2x - 3$

when $y = 0$

$x = -3$ or $1$

when $x = 0$

$y = -3$

b) $y = 3x - 1$

$y = (x+3)(x-1)$

$= x^2 + 2x - 3$

$\Rightarrow 3x - 1 = x^2 + 2x - 3$

$x^2 - x - 2 = 0$

c) $x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1$, or $2$

$y = 3x - 1$

$y = -4$ or $5$

Coordinates are $(-1, -4)$ and $(2, 5)$

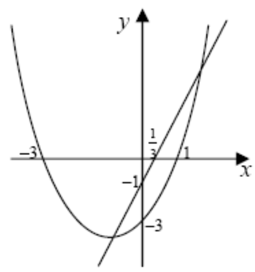
### ***Commentary***

(a) It was a sensible idea to start the solution at the top of page 2 of the booklet, rather than on the few lines on the front cover. The parabola has all the main features and scores full marks. The candidate fails to indicate the intercept of the straight line on the x-axis and loses a mark. This mark could have been scored if a statement that "$x=\frac{1}{3}$ when y=0 " had appeared alongside the sketch where the candidate has clearly explained how to find the intercepts for the quadratic curve.

(b) Sufficient working is shown here to score full marks and the proof is set out clearly. Many candidates forgot to include "=0" and lost the mark.

(c) The quadratic is factorised correctly and the values of x are stated clearly. The candidate uses the equation of the straight line to find the coordinates of y and the coordinates of the points of intersection are written down in the final line of the solution. This is a good exemplar which candidates would be wise to follow.

### ***Mark scheme***

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)** | $L$: straight line with positive gradient and negative intercept on $y$-axis | B1 | | Line must cross both axes but need not reach the curve |
| | cutting at $\left(\frac{1}{3},0\right)$ and $(0,-1)$ (intercepts **stated** or marked on sketch) | B1 | | Condone 0.33 or better for $\frac{1}{3}$ |
| | $C$: **attempt at parabola** $\cup$ or $\cap$ through $(-3,0)$ and $(1,0)$ or values $-3$ and 1 stated as intercepts on $x$-axis | B1 | |  |
| | $\cup$ shaped graph – vertex below $x$-axis and cutting $x$-axis twice | M1 | | |
| | through $(0,-3)$ and minimum point to left of $y$-axis | A1 | 5 | ($y$-intercept or coordinates marked) |
| **(b)** | $(x+3)(x-1)=3x-1$ | M1 | | |
| | $x^2+3x-x-3-3x+1=0$ | | | |
| | $\Rightarrow x^2-x-2=0$ | A1 | 2 | AG; must have "= 0" and no errors |
| **(c)** | $(x-2)(x+1)=0$ | M1 | | $(x\pm1)(x\pm2)$ or use of formula (one slip) |
| | $\Rightarrow x=2,\ -1$ | A1 | | correct values imply M1A1 |
| | Substitute one value of $x$ to find $y$ | m1 | | |
| | Points of intersection (2, 5) and (–1,–4) | A1 | 4 | May say $x=2, y=5$ etc<br>SC: (2, 5) $\Rightarrow$ B2<br>$(-1,-4) \Rightarrow$ B2 without working |
| | **Total** | | **11** | |

***Question 2***

| | | |
|---|---|---|
| **2** | It is given that $x = \sqrt{3}$ and $y = \sqrt{12}$. | |
| | Find, in the simplest form, the value of: | |
| | (a) $xy$ ; | *(1 mark)* |
| | (b) $\dfrac{y}{x}$ ; | *(2 marks)* |
| | (c) $(x+y)^2$ . | *(3 marks)* |

***Student response***

2\. a) $x = \sqrt{3}$ and $y = \sqrt{12}$

$$\sqrt{3} \times \sqrt{12} = \sqrt{3} \times 2\sqrt{3}$$

$$= 3\sqrt{3}$$

✗

b) $$\frac{\sqrt{12}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$$

✓

$$\sqrt{3} \times \sqrt{3} = 2\sqrt{3} \rightarrow \sqrt{12}$$

2

c) $(\sqrt{3} + \sqrt{12})^2$

M1

| | $\sqrt{3}$ | $\sqrt{12}$ |
|---|---|---|
| $\sqrt{3}$ | 3 | $3\sqrt{3}$ |
| $\sqrt{12}$ | $3\sqrt{3}$ | 12 |

$$3 + 3\sqrt{3} + 3\sqrt{3} + \sqrt{12}$$

A0

$$= 15 + 6\sqrt{3}$$

***Commentary***

This solution illustrates a common error where this candidate confuses multiplication with addition. Rather than obtaining an answer of $\sqrt{36} = 6$ for part (a), the two surds are added to give $3\sqrt{3}$. This same error is evident in part (c) in the grid being used as working and so only a method mark is scored for attempting to multiply out two brackets. The candidate answers part (b) correctly where the division of two surds is required.

## *Mark Scheme*

| | | | | |
|---|---|---|---|---|
| **2(a)** | $xy = 6$ | B1 | 1 | B0 for $\sqrt{36}$ or $\pm 6$ |
| **(b)** | $\frac{y}{x} = \frac{2\sqrt{3}}{\sqrt{3}}$ or $\sqrt{\frac{12}{3}}$ or $\sqrt{\frac{4}{1}}$ or $\frac{\sqrt{12}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ | M1 | | Allow M1 for $\pm 2$ |
| | $= 2$ | A1 | 2 | |
| **(c)** | $x^2 + 2xy + y^2$ or $(\sqrt{3} + 2\sqrt{3})^2$ correct | M1 | | or $\left(\sqrt{3}+\sqrt{12}\right)\left(\sqrt{3}+\sqrt{12}\right)$ expanded as 4 terms – no more than one slip |
| | **Correct** with 2 of $x^2, y^2, 2xy$ simplified | A1 | | Correct but unsimplified – one more step |
| | $3 + 2\sqrt{36} + 12$ or $3^2 \times 3$ or $\left(3\sqrt{3}\right)^2$ | | | |
| | $= 27$ | A1 | 3 | |
| | **Total** | | **6** | |

***Question 3***

**3** Two numbers, $x$ and $y$, are such that $3x + y = 9$, where $x \geqslant 0$ and $y \geqslant 0$.

It is given that $V = xy^2$.

(a) Show that $V = 81x - 54x^2 + 9x^3$. *(2 marks)*

(b) (i) Show that $\dfrac{dV}{dx} = k(x^2 - 4x + 3)$, and state the value of the integer $k$. *(4 marks)*

(ii) Hence find the two values of $x$ for which $\dfrac{dV}{dx} = 0$. *(2 marks)*

(c) Find $\dfrac{d^2V}{dx^2}$. *(2 marks)*

(d) (i) Find the value of $\dfrac{d^2V}{dx^2}$ for each of the two values of $x$ found in part (b)(ii). *(1 mark)*

(ii) Hence determine the value of $x$ for which $V$ has a maximum value. *(1 mark)*

(iii) Find the maximum value of $V$. *(1 mark)*

***Student Response***

3a $3x + y = 9 \quad x \geq 0 \quad y \geq 0$

$V = xy^2$

$x = \frac{9-y}{3}$

~~dV~~ $y = 9 - 3x$

$y^2 = (9-3x)^2$

$= 81 - \cancel{3}27x - 27x + 9x^2$

$= 81 - \cancel{\phantom{2}}\, 54x + 9x^2$

$x = \frac{9-(9-3x)}{3}$

$= 3 - (3 - x)$

$= x$

$x(81 - 54x + 9x^2) = xy^2$

$81x - 54x^2 + 9x^3 = xy^2$

b.i. $81x - 54x^2 + 9x^3$ M1

$= 9(9x - 6x^2 + x^3)$ A2

$\frac{dV}{dx} = 9(9 - 12x + 3x^2)$ A0

$= \cancel{8B32k}\ 3(\cancel{3}x^2 - 4x + 3)$

$k = 3$

ii. $3(x^2 - 4x + 3) = 0$

$3(x-3)(x-1) = 0$

Question number

$x = 1$ or $x = 3$ $\qquad 3(x^2 - 4x + 3)$

c. $\dfrac{d^2V}{dx^2} = 3(2x - 4)$ $\qquad$ M1

$= 6x - 12$ $\qquad$ A0

di. when $x = 1$

$\dfrac{d^2V}{dx^2} = 6(1) - 12$

$= -6$

when $x = 3$

$\dfrac{d^2V}{dx^2} = 6(3) - 12$

$= 6$

ii $x$ is a maximum at $x = 1$

iii 1

***Commentary***

(a) Many candidates made heavy weather of simplifying the expression for $V$ to obtain the printed answer. Rather than leaving $x$ alone, the candidate rearranges various formulae to get back to x as a multiplier. Nevertheless, the algebra is sound and the candidate scores full marks.

(b)(i) Having found the correct derivative, many candidates were unable to find the correct value of $k$. Here the candidate believes the value of $k$ is 3 instead of 27.
(ii) Because the value of $k$ was often incorrect, the mark scheme allowed for candidates to make this slip and not lose too many more marks. Full marks were awarded for finding the correct values of $x$.
(c) Again a method mark was awarded even though the candidate had changed the value of $\dfrac{dV}{dx}$, because of an incorrect value of $k$, and whatever the candidate wrote down as their second derivative was used to award marks in part (d).
(d) The values of $\dfrac{d^2V}{dx^2}$ are consistent with the candidate's second derivative and are credited. The value $x$=1 also scores the mark for indicating the value of $x$ when the maximum occurs, but the candidate fails to substitute $x$=1 into the expression for $V$ and so does not earn the final mark.

*Mark Scheme*

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **3(a)** | $V = x(9-3x)^2$ | M1 | | Attempt at $V$ in terms of $x$ (condone slip when rearranging formula for $y = 9 - 3x$) or $(9-3x)^2 = 81 - 54x + 9x^2$ |
| | $V = x(81-54x+9x^2)$ | | | |
| | $= 81x - 54x^2 + 9x^3$ | A1 | 2 | AG; no errors in algebra |
| **(b)(i)** | $\frac{dV}{dx} = 81 - 108x + 27x^2$ | M1<br>A1<br>A1 | | One term correct<br>Another correct<br>All correct (no + $c$ etc) |
| | $= 27(x^2 - 4x + 3)$ | A1 | 4 | CSO; all algebra and differentiation correct |
| **(ii)** | $(x-3)(x-1)$ or $(27x-81)(x-1)$ etc<br>$\Rightarrow x = 1, 3$ | M1<br>A1 | 2 | "Correct" factors or correct use of formula<br>SC: B1,B1 for $x = 1$, $x = 3$ found by inspection (provided no other values) |
| **(c)** | $\frac{d^2V}{dx^2} = -108 + 54x$ (condone one slip) | M1<br>A1 | 2 | ft their $\frac{dV}{dx}$ (may have cancelled 27 etc)<br>CSO; all differentiation correct |
| **(d)(i)** | $x = 3 \Rightarrow \frac{d^2V}{dx^2} = 54$; $x = 1 \Rightarrow \frac{d^2V}{dx^2} = -54$ | B1√ | 1 | ft their $\frac{d^2V}{dx^2}$ and their **two** $x$-values |
| **(ii)** | ($x =$) 1 (gives maximum value) | E1 | 1 | Provided their $\frac{d^2V}{dx^2} < 0$ |
| **(iii)** | $V_{max} = 36$ | B1 | 1 | CAO |
| | **Total** | | **13** | |

***Question 4***

| | | |
|---|---|---|
| **4** | (a) | Express $x^2 - 3x + 4$ in the form $(x-p)^2 + q$, where $p$ and $q$ are rational numbers. *(2 marks)* |
| | (b) | Hence write down the minimum value of the expression $x^2 - 3x + 4$. *(1 mark)* |
| | (c) | Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 - 3x + 4$. *(3 marks)* |

***Student Response***

4a) $x^2 - 3x + 4 = (x - 1\frac{1}{2})^2 - (1\frac{1}{2})^2 + 4$

$\frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = 2\frac{1}{4}$

$(x - \frac{3}{2})^2 - 2\frac{1}{4} + 4$

$\Rightarrow x^2 - 3x + 4 = (x - 1\frac{1}{2})^2 + 1\frac{3}{4}$

or $(x - \frac{3}{2})^2 + \frac{7}{4}$ 2

b) Minimum value when $x = 1\frac{1}{2}$ 0

c) A translation along the vector $\begin{pmatrix} 1\frac{1}{2} \\ 1\frac{3}{4} \end{pmatrix}$ 3

(5)

***Commentary***

(a) The candidate provides an excellent solution to this part of the question. Not many found the correct values of both *p* and *q* . The fractional value of *p* caused problems to many, but this candidate carefully squared $\frac{3}{2}$ and realised the need to subtract this value from 4 to obtain $q = \frac{7}{4}$.

(b) Many candidates did not understand the term “ value of the expression” ; some gave the coordinates of the minimum point ; this candidate gave the *x*- value rather than the correct value of $\frac{7}{4}$.

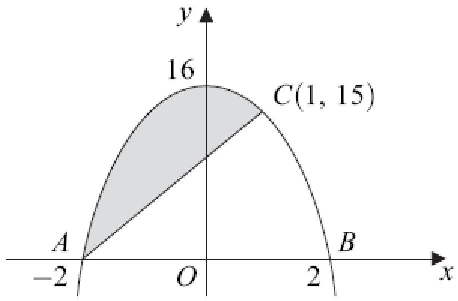
(c) The correct word “translation” is used and this is described by the correct vector and so full marks are scored. It would be wise if all candidates learnt how to express such a transformation in this concise way.

## *Mark Scheme*

| | | | | |
|---|---|---|---|---|
| **4(a)** | $\left(x-\frac{3}{2}\right)^2$ | B1 | | Must have $(\ )^2$ $p=1.5$ |
| | $+\frac{7}{4}$ | B1 | 2 | $q=1.75$ |
| **(b)** | Minimum value is $\frac{7}{4}$ | B1$\surd$ | 1 | ft their $q$ or correct value |
| **(c)** | **Translation**<br>(and **no other** transformation stated) | E1 | | (**not** shift, move, transformation etc) |
| | through $\begin{bmatrix}\frac{3}{2}\\ \frac{7}{4}\end{bmatrix}$ (or equivalent in words) | M1<br>A1 | 3 | M1 for one component correct or ft their $p$ or $q$ values<br>CSO; condone 1.5 right and 1.75 up etc |
| | **Total** | | **6** | |

***Question 5***

**5** The curve with equation $y = 16 - x^4$ is sketched below.




The points $A(-2, 0)$, $B(2, 0)$ and $C(1, 15)$ lie on the curve.

(a) Find an equation of the straight line $AC$. *(3 marks)*

(b) (i) Find $\int_{-2}^{1} (16 - x^4)\, dx$. *(5 marks)*

(ii) Hence calculate the area of the shaded region bounded by the curve and the line $AC$. *(3 marks)*

***Student Response***

b) i) $\int_{-2}^{1} (16 - x^4)\,dx$

$= 16x - \frac{1}{5}x^5$

$= \left(16x - \frac{1}{5}x^5\right)_{-2}^{1}$

$= (16(1) - \frac{1}{5}(1)^5) - (16(-2) - \frac{1}{5}(-2)^5)$

$-2$
$4$
$-8$
$16$
$-32$

$= (16 - \frac{1}{5}) - (-32 + \frac{32}{5})$

$= 16 - \frac{1}{5} + 32 - \frac{32}{5}$

$= 48 - \frac{33}{5}$

$= \frac{240}{5} - \frac{33}{5}$

$= \frac{207}{5}$

$50 \times 5$

$250 - 10$

$240$

$\frac{48}{5}$
$240$

ii) area of shaded region = $\int$ − area of triangle

$\frac{207}{5} - (0.5 \times 3 \times 15)$

$= \frac{207}{5} - \frac{45}{2}$

$\frac{45}{5}$
$225$

$= \frac{414}{10} - \frac{225}{10}$

$= \frac{189}{10}$ or $18.9$

$409 - 220$
$9$
$229$
$80$
$309$
$100$

.

***Commentary***

This candidate has produced a good solution for part (b) of the question. Many candidates made arithmetic errors when handling fractions, but in the example above, the correct use of brackets has helped to produce accurate work. It is interesting to see how $(-2)^5$ has been calculated alongside the main body of working.
The fractions caused problems for many but this candidate sets the method out clearly and avoids mistakes. Those candidates who did not identify the correct triangle scored no marks in part (b)(ii) , but here the base is 3 and the height is 5.

**Mark Scheme**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5(a)** | Grad $AC = \frac{15}{3} = 5$ | B1 | | OE |
| | Equation of $AC$: $y = m(x+2)$ or $(y-15) = m(x-1)$ | M1 | | Or use of $y = mx + c$ with $(-2, 0)$ or $(1, 15)$ correctly substituted for $x$ and $y$ |
| | $y = 5x + 10$ | A1 | 3 | OE eg $y - 15 = 5(x-1)$, $y = 5(x+2)$ |
| **(b)(i)** | $\left[16x - \frac{x^5}{5}\right]$ | M1<br>A1<br>A1 | | Raise one power by 1<br>One term correct<br>All correct |
| | $\left(16 - \frac{1}{5}\right) - \left(-32 + \frac{32}{5}\right)$ | m1 | | $\mathrm{F}(1) - \mathrm{F}(-2)$ attempted |
| | $= 41\frac{2}{5}$ (or 41.4 , $\frac{207}{5}$ etc) | A1 | 5 | CSO; withhold if $+c$ added |
| **(ii)** | Area $\Delta = \frac{1}{2} \times 3 \times 15$ or $22\frac{1}{2}$ or 22.5 | B1 | | Or $\int_{-2}^{1} (5x+10)\,\mathrm{d}x = 22.5$ |
| | Shaded area = "their (b)(i) answer" – correct triangle | M1 | | Condone "difference" if $\Delta > \int$ |
| | $\Rightarrow$ shaded area $= 18\frac{9}{10}$ | A1 | 3 | CSO; OE (18.9 etc) |
| | **Total** | | **11** | |

***Question 6***

**6** The polynomial $p(x)$ is given by $p(x) = x^3 + x^2 - 8x - 12$.

(a) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 1$. *(2 marks)*

(b) (i) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as the product of linear factors. *(3 marks)*

(c) (i) The curve with equation $y = x^3 + x^2 - 8x - 12$ passes through the point $(0, k)$. State the value of $k$. *(1 mark)*

(ii) Sketch the graph of $y = x^3 + x^2 - 8x - 12$, indicating the values of $x$ where the curve touches or crosses the $x$-axis. *(3 marks)*

***Student Response***

6. $p(x) = x^3 + x^2 - 8x - 12$

| | $x^2$ | $+2x$ | $-6$ |
|---|---|---|---|
| $x$ | $x^3$ | $+2x^2$ | $-6x$ |
| $-1$ | $-x^2$ | $-2x$ | $+6$ |

$x^2$ $\quad -8x \quad -12$

remainder = 18

$p(1) = 1^3 + 1^2 - 8 \times 1 - 12$
$= -6 - 12$
$= -18$

M1 A0

b) i)

$p(-2) = -2^3 + -2^2 - 8\times -2 - 12$

$= -8 + 4 + 16 - 12$

$= 0$ ∧ M1 A0

ii)

| | $x^2$ | $-x$ | $-6$ |
|---|---|---|---|
| $x$ | $x^3$ | $-x^2$ | $-6x$ |
| $+2$ | $2x^2$ | $-2x$ | $-12$ |

$(x+2)(x^2 - x - 6)$

$(x+2)(x+2)(x-3)$ ✓

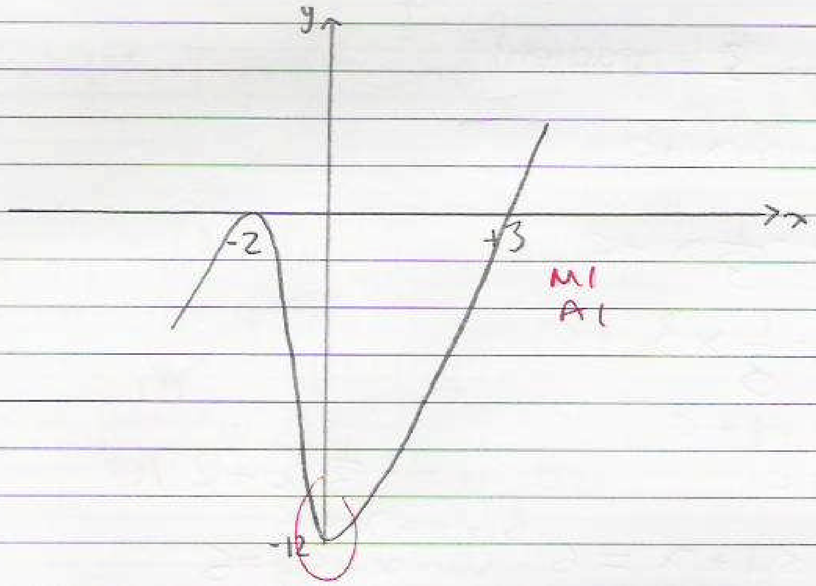
c i)

$k = 0^3 + 0^2 - 8\times 0 - 12$

$k = -12$ ✓

ii)

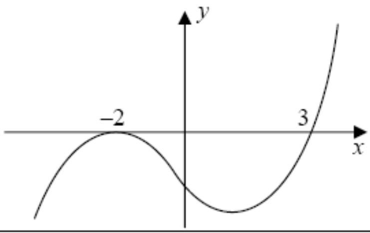
$x = -2, +3, -2$



***Commentary***

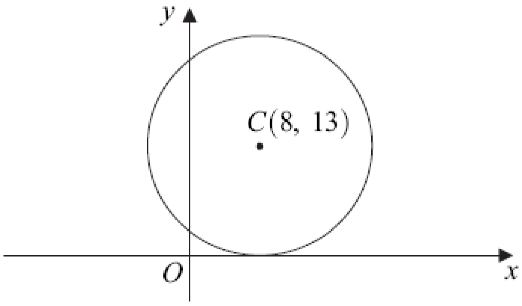
(a)The work in the grid was regarded as additional working and a method mark was awarded for finding p(1) which was correctly evaluated as –18. Had the answer been left as such the candidate would have scored full marks, but the comment “remainder =18” loses the A mark.
(b) Although p(–2) is evaluated and shown to equal 0, again a mark is lost for not completing the proof and saying that x+2 is a factor. The factorisation is correct and scores full marks.
(c)Thevalue of *k* is found correctly to equal –12 and this value is shown on the sketch. Because p(1) was earlier shown to equal –18, this information was expected to be used when sketching the curve and so the minimum point should have been shown to the right of the y-axis. Many candidates produced a sketch similar to this which earned 2 out of the 3 marks.

***Mark Scheme***

| | | | | |
|---|---|---|---|---|
| **6(a)** | Remainder $= \mathrm{p}(1) = 1+1-8-12$<br>$= -18$ | M1<br>A1 | 2 | Use of $\mathrm{p}(1)$ NOT long division |
| **(b)(i)** | $\mathrm{p}(-2) = -8+4+16-12$<br>$= 0 \Rightarrow (x+2)$ is factor | M1<br>A1 | 2 | NOT long division<br>$\mathrm{p}(-2)$ shown $= 0$ **and** statement |
| **(ii)** | Quad factor by comparing coefficients or $\left(x^2+kx\pm 6\right)$ by inspection | M1 | | Or full long division or attempt at Factor Theorem using $\mathrm{f}(\pm 3)$ |
| | $\mathrm{p}(x) = (x+2)(x^2-x-6)$ | A1 | | Correct quadratic factor or $(x-3)$ shown to be factor by Factor Theorem |
| | $\mathrm{p}(x) = (x+2)^2(x-3)$ or<br>$(x+2)(x+2)(x-3)$ | A1 | 3 | CSO; SC: B1 for $(x+2)(x***)(x-3)$ by inspection or without working |
| **(c)(i)** | $(k=)\ -12$ | B1 | 1 | Condone $y=-12$ or $(0,-12)$ |
| **(ii)** |  | M1<br>A1<br>A1 | 3 | Cubic shape (one max and one min)<br>Maximum at $(-2,0)$ and through $(3, 0)$ – at least one of these values marked<br>“correct” graph as shown (touching smoothly at –2, 3 marked and minimum to right of $y$-axis) |
| | **Total** | | **11** | |

***Question 7***

**7** The circle $S$ has centre $C(8, 13)$ and touches the $x$-axis, as shown in the diagram.




(a) Write down an equation for $S$, giving your answer in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(2 marks)*

(b) The point $P$ with coordinates $(3, 1)$ lies on the circle.

(i) Find the gradient of the straight line passing through $P$ and $C$. *(1 mark)*

(ii) Hence find an equation of the tangent to the circle $S$ at the point $P$, giving your answer in the form $ax + by = c$, where $a$, $b$ and $c$ are integers. *(4 marks)*

(iii) The point $Q$ also lies on the circle $S$, and the length of $PQ$ is 10. Calculate the shortest distance from $C$ to the chord $PQ$. *(3 marks)*

***Student Response***

$$\begin{array}{r} 169 \\ +\ 64 \\ \hline 233 \end{array} \quad \begin{array}{r} 13 \\ \times 13 \\ \hline 39 \\ +\ 130 \\ \hline 169 \end{array} \quad \begin{array}{r} 8 \\ \times 8 \\ \hline 64 + 25 \\ 144 \\ \hline 169 \end{array}$$

7a) C(8, 13)

$(x-8)^2 + (y-13)^2 - 64 - 169 = 0$

$(x-8)^2 + (y-13)^2 - 233 = 0$ R1

$(x-8)^2 + (y-13)^2 = \sqrt{233}.$ B0 (116.5)

bi) C(8, 13) P(3, 1)

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1-13}{3-8.} = \frac{-12}{-5} = \frac{12}{5}$ ✓

bii) $\frac{y-1}{x-3} = \frac{12}{5}$ X M0

$5(y-1) = 12(x-3)$

$5y - 5 = 12x - 36.$

$5y = 12x - 31$

$-12x + 5y = -31$

$12x - 5y = 31$

biii) PQ = 10 C(8,13) P(3, 1)

$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(3-8)^2 + (1-13)^2}$

$= \sqrt{5^2 + 12^2}$

$= \sqrt{25 + 144}$

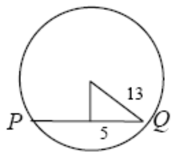
$\sqrt{169}$

$= 13.$ ^

***Commentary***

(a)This solution illustrates a very common error when finding the equation of the circle. The candidate failed to realise that, because the circle touches the x-axis, the radius is 13. Instead, some attempt is made to complete the squares and hence the value of 233 is obtained, instead of the correct value of 169; a square root sign is added for good measure.
(b)(i) As with most candidates, the gradient is found correctly.
(b)(ii)The candidate fails to realise that the perpendicular gradient is required to find the equation of the tangent and hence no marks are scored here.
(b)(iii)The correct radius is found in this part, but this should have been evident from the diagram in the question. It was necessary to use Pythagoras' Theorem with the length of half the chord and the length of the radius so as to obtain the distance from the centre of the circle to the midpoint of the chord.

***Mark Scheme***

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(a)** | $(x-8)^2+(y-13)^2$ | B1 | | **Exactly this** with + and squares |
| | $=13^2$ | B1 | 2 | Condone 169 |
| **(b)(i)** | grad $PC=\frac{12}{5}$ | B1 | 1 | Must simplify $\frac{-12}{-5}$ |
| **(ii)** | grad of tangent $=\frac{-1}{\text{grad } PC}=-\frac{5}{12}$ | B1√ | | Condone $-\frac{1}{2.4}$ etc |
| | tangent has equation $y-1=-\frac{5}{12}(x-3)$ | M1<br>A1 | | ft gradient but M0 if using grad $PC$<br>Correct – but not in required final form |
| | $5x+12y=27$ OE | A1 | 4 | MUST have integer coefficients |
| **(iii)** | half chord $=5$ | B1 | | Seen or stated |
| |   $d^2=(\text{their } r)^2-5^2$ (provided $r>5$) | M1 | | Pythagoras used correctly $d^2=13^2-5^2$ |
| | Distance $=12$ | A1 | 3 | CSO |
| | **Total** | | **10** | |

***Question 8***

**8** The quadratic equation $(k+1)x^2 + 4kx + 9 = 0$ has real roots.

(a) Show that $4k^2 - 9k - 9 \geqslant 0$. *(3 marks)*

(b) Hence find the possible values of $k$. *(4 marks)*

***Student Response***

8a. When an equation has real roots,

$b^2 - 4ac \geqslant 0$

$(4k)^2 - 4(k+1)(9) \geqslant 0$ ✓

$16k^2 - 36k \cancel{+} - 36 \geqslant 0$ ✓

$4k^2 - 9k - 9 \geqslant 0$

b. ~~(2k ... )(2k ... ) ≥ 0~~

$(4k+3)(k-3) \geqslant 0$ M1

A1

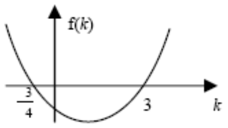
~~M0~~

$-3/4 \leqslant k \leqslant 3$ ✗

***Commentary***

(a) This is a good solution to this part of the question. Many candidates did not use brackets and casually wrote things such as "$4k^2 = 16k^2$" and lost a mark. The candidate wisely writes a general condition for real roots in terms of *a,b* and *c* and then applies this immediately to the discriminant expression in terms of *k.* Those candidates who simply inserted the greater than or equal to sign on the last line did not convince examiners they were doing anything other than copying the answer printed in the question paper. On line 4 of the solution the candidate writes a + sign but clearly obliterates this and writes a minus sign after removing the brackets. This is good practice because some candidates change a plus sign into a minus sign by writing the horizontal bar a little bolder and examiners are not always convinced what the candidate's intended sign actually is.

(b) This candidate would have been wise to have drawn a sketch of $y = (4k+3)(k-3)$ or used a sign diagram with the critical points $-\frac{3}{4}$ and 3 . This might then have earned another method mark and could possibly have prompted the correct final inequality. The marks awarded here are M1 for factorising correctly and A1 for finding the correct critical values as seen in the final inequality, even though it is incorrect.

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **8(a)** | $b^2-4ac=16k^2-36(k+1)$ | M1 | | Condone one slip |
| | Real roots: discriminant $\geqslant 0$ | B1 | | |
| | $\Rightarrow 16k^2-36k-36\geqslant 0$ | | | |
| | $\Rightarrow 4k^2-9k-9\geqslant 0$ | A1 | 3 | AG (watch signs) |
| **(b)** | $(4k+3)(k-3)$ | M1 | | Or correct use of formula (unsimplified) |
| | critical points $(k=)\ -\frac{3}{4}\ ,\ 3$ | A1 | | Not in a form involving surds<br>Values may be seen in inequalities etc |
| |  sketch | M1 | | Or sign diagram |
| | $k\geqslant 3,\quad k\leqslant -\frac{3}{4}$ | A1 | 4 | NMS full marks<br>Condone use of word "and" but final answer in a form such as $3\leqslant k\leqslant -\frac{3}{4}$ scores A0 |
| | **Total** | | 7 | |